Number II Year 9

Fractions

A fraction is just a number with an uncompleted division, nothing more.

$\frac{2}{5} = 2 \div 5$

A fraction can be formed whenever we have one number "out of" some total.

"8 out of 13" gives a fraction of $\frac{8}{13}$

To find a fraction of something, we multiply by that fraction.

Three-quarters of 15 = $\frac{3}{4} \times 15 = \frac{45}{4}$

Any decimal can be converted to a fraction by placing the decimal portion over the power of ten for the last decimal place.

$3.34 = 3\,\frac{34}{100}$ or $\frac{334}{100}$

The form of a fraction can be changed by multiplying top and bottom by a constant. A calculator will do this automatically in order to give the simplest form.

$\frac{3}{4} = \frac{3}{4}\,\frac{\times 5}{\times 5} = \frac{15}{20}$ and $\frac{8}{12} = \frac{8}{12}\,\frac{\div 4}{\div 4} = \frac{2}{3}$

A mixed number (whole number + fraction) can be converted to an improper fraction (one where the denominator is larger) by converting the whole number to fractional form and adding it in.

$2\,\frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$ and $\frac{18}{5} = \frac{15+3}{5} = 3\,\frac{3}{5}$

The key for most fraction questions is identifying the number which the fraction is operating on.

e.g. If $\frac{1}{3}$ a cake is eaten, and Bill eats $\frac{1}{4}$ *of what is left*, he is eating $\frac{1}{4}$ of the $\frac{2}{3}$ left = $\frac{1}{6}$

That means there is $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$ of the cake left.

but if there is a cake and Sam eats $\frac{1}{3}$ and Bill eats $\frac{1}{4}$, Bill is still eating $\frac{1}{4}$ a whole cake.

That means there is $\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ of the cake left.

The skills for achieving in NCEA Level 1 are being able to form fractions and find a fraction of something in the context of a question, and students should concentrate on those skills first. The arithmetic will be taken care of by the calculator, provided they learn how to use the fraction button on their calculator.

Fraction Operations by Hand

For students who hope to go on to higher level Maths at Years 12 and 13, the ability to do fraction questions by hand becomes important and is worth learning early.

Fractions must have the same denominator before they are added or subtracted.

$$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

To multiply the top and bottom are multiplied separately. Use the improper form if greater than 1.

$$\frac{3}{4} \times 1\frac{1}{4} = \frac{3}{4} \times \frac{5}{4} = \frac{15}{20} \left(= \frac{3}{4}\right)$$

To divide, the divisor is inverted (flipped) first then multiplied.

$$\frac{3}{5} \div 3\frac{2}{5} = \frac{3}{5} \div \frac{17}{5} = \frac{3}{5} \times \frac{5}{17} = \frac{15}{85} \left(= \frac{3}{17}\right)$$

In general students should get used to writing fractions in improper form and not feel the need to convert them to mixed numbers every time. Improper fractions are the correct form for algebraic equations.

A whole number can be replaced by itself over 1, which is particularly needed for division.

$$\frac{3}{4} \div 2 = \frac{3}{4} \div \frac{2}{1} = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

Percentages

Percentage questions are a form of fraction question, answered basically the same way.

A percentage is just a fraction out of 100.

$22\% = \frac{22}{100}$ and so likewise $\frac{145}{100} = 145\%$

A percentage can be formed whenever we have one number "out of" some total by forming the fraction and converting it via the decimal.

"8 out of 13" gives a fraction of $\frac{8}{13} = 8 \div 13 = 0.61538 = \frac{61.538}{100} = 61.54\%$

A percentage of something, means we multiply, after we convert it into the appropriate fraction.

65% of $15 = \frac{65}{100} \times 15 = 9.75$

A percentage change is always measured as a fraction of change from the **start** value.

If \$55 rises to \$60, the % change is $= \frac{\text{change}}{\text{start}} = \frac{5}{55} = 5 \div 55 = 0.0909 = \frac{9.0909}{100} = 9.09\ \%$

A percentage increase can be found by adding the percentage to the original value.

To increase 50 by 20% $= 50 + \frac{20}{100} \times 50 = 50 + 10 = 60$

This can be shortened to one step once students are confident, by adding the original value (100%) to the increased %.

50 increased by 20% = 120% of 50 $= \frac{120}{100} \times 50 = 60$

A percentage decrease can be found by subtracting the change from the original.

To decrease 50 by 20% $= 50 - \frac{20}{100} \times 50 = 50 - 10 = 40$

This can be shortened to one step as well, by taking the reduction from 100 % as the desired value.

To decrease 50 by 20% = 80% of 50 $= \frac{80}{100} \times 50 = 40$

To calculate backwards from a known end point to the starting value requires using algebraic methods.

If a value is increased by 25% and ends up being \$400, then 125% of x = \$400

This rearranges to give $x = 400 \div 1.25$ (being $\frac{125}{100}$), so x = \$320

(Note: if you work out 25% of \$400 and subtract it you get \$300, which is wrong.)

Ratios

Ratios are not part of the main Year 9 syllabus, but some knowledge of them is useful.

> A ratio is a way of writing a relationship between two (or more) quantities.
>
> A : B of 2 : 1 means there are two As for every B.
>
> A ratio can be converted to relative fractions of the whole, by using the **sum** as the denominator.
>
> C : D of 3 : 4 means C is $\frac{3}{7}$ of the total, and D is $\frac{4}{7}$ of the total

The " : " is usually said, and sometimes written, "to". Sometimes the amounts are referred to as "parts", so that X : Y of 2 : 3 can be described as "two parts of X to three parts of Y".

A ratio can be simplified to its simplest terms in exactly the same way as a fraction, by dividing by the highest common factor.

$50 : 30 = 5 \times 10 : 3 \times 10 = 5 : 3$

Many ratio questions are answered by multiplying up a ratio by the relevant simplest form in the reverse process to simplifying.

If A : B is 3 : 2, and there are 72 As, then our multiplication factor is 24 (= 72 ÷ 3)

$3 : 2 = 3 \times 24 : 2 \times 24 = 72 : 48$, so there must be 48 Bs.

A sum is divided into a ratio by finding the total of the ratio, and using the total number of "parts" and so finding how large each part is.

To divide \$60 in the ratio of 3 : 2, we see that 2 + 3 = 5, so each part is \$60 ÷ 5 = \$12

Now we can multiply up: $3 : 2 = 3 \times 12 : 2 \times 12$, and our sum is divided \$36 : \$24

The methods remain the same if the ratio has three or more parts.

To divide \$180 in the ratio of 6 : 2 : 1, we take 6 + 2 + 1, so each part is \$180 ÷ 9 = \$ 20

So our sum is divided $6 \times 20 : 2 \times 20 : 1 \times 20$ which is \$120 : \$40 : \$20

A true ratio has no units given, but is true for all units.

If E :F is 6 : 1, then for each 6 grams of E there will be one gram of F

but equally for 6 tonnes of E there will be one tonne of F

Conversions

To convert from the units with the 1 in the conversion ratio to the other units, we multiply.

1 inch = 2.54 cm. So 3 inches = 3 × 2.54 = 7.62 cm

To convert to the units with the 1 in the conversion ratio to the other units, we divide.

1 inch = 2.54 cm. So 3 cm = 3 ÷ 2.54 = 1.18 inches

It is often best to draw this on the paper to be certain to get the right operation. The direction to get the multiplication is fairly obvious, and division is the opposite of multiplication.

× 1.61

1 mile = 1.61 km

÷ 1.61

so 10 miles = 10 × 16.09 = 16.1 km

The multiplication is always away from the 1. Nothing changes if the other figure is smaller than 1.

× 0.62

0.62 miles = 1 km

÷ 0.62

so 16 miles = 16 ÷ 0.62 = 25.8 km

If the conversion factor is not given to a ratio of one, a division by either side will convert it to that form, and then you can work as before.

If NZ$120 = US$90, then NZ$ $\frac{120}{120}$ = US$ $\frac{90}{120}$, which is NZ$1 = US$0.75

Proportion and Rates

Proportion and rate questions (like the related ratio and conversion questions) always use multiplication and division, not addition and subtraction.

Recipe type questions can be answered by methods basically the same as those for ratios, the only difference being that a recipe may not have each part with the same units.

Recipe for 12 meatballs:	500g hamburger
	1 cup minced onion
	1 egg
	¼ cup milk
	¼ cup bread crumbs
	salt and pepper

To make 300 meatballs, would require $\frac{300}{12} = 25 \times$ as much of each ingredient.

The general method for proportion questions is to divide to find how much a single unit of interest takes, then multiply it up afterwards.

Bill runs 8 km in 36 minutes.

So he takes 36 ÷ 8 = 4.5 min to run 1 km.

At the same speed he would run 7 km in 7 × 4.5 = 31.5min (= 31 m 30s)

8 km →(÷ 8) 1 km →(× 7) 7 km

35 min →(÷ 8) 4.5 min →(× 7) 31.5 min

Rate is the general mathematical word for the speed at which some amount is changing. If we want the average rate we use $\frac{\text{amount changed}}{\text{time taken}}$.

If Hamilton's population grows from 140,000 to 150,000 in four years

the rate of change is $\frac{(150{,}000 - 140{,}000)}{4}$ = 2,500 people per year.

Students need to be particularly comfortable with calculating average speed, which is the rate of change of distance with time. Speed = $\frac{\text{distance}}{\text{time}}$

If a car travels 160 km in 1½ hours, then its average speed = 160 ÷ 1.5 = 107 km/hr