Trigonometry Year 10

Trigonometry is mostly about following a set of rules, which need to be memorised. No amount of working will help if the rules have been memorised poorly.

The steps for any trigonometry problem are always the same:

1. Find a right angle triangle to solve.
2. Name the sides using the conventions below.
3. If no angle is involved, used Pythagoras' Theorem, otherwise $S\frac{O}{H}\quad C\frac{A}{H}\quad T\frac{O}{A}$
4. Choose the formula that gives uses the information that you have been given.
5. Put the values into the formula and solve.
6. Make sure the answer looks right.

If the triangle does not have a 90°, then you cannot use the rules of trigonometry given here. Instead the problem must be converted into one where there is one, or more, right angle triangles. This often involves some geometry.

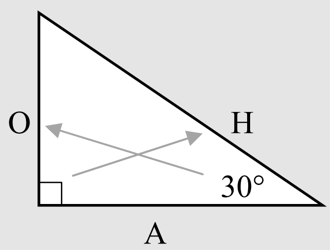
Naming Convention

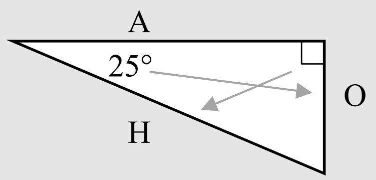
Naming the sides correctly is key to Trigonometry, as picking the correct formulas depends on it.

The long side of any right angle triangle is called the Hypotenuse.

The hypotenuse is always the side across from, and not touching, the right angle.

If another angle is given or wanted, then the side opposite that is called the Opposite, and the side touching it is called the Adjacent.





The Hypotenuse can be the top, bottom, left or right side: what matters is that it is always opposite the right angle.

Always label the Hypotenuse first. This prevents confusion about which is the Adjacent, and makes sure that questions are not attempted on triangles without a right angle.

Pythagoras Theorem

For any right angled triangle: the long side squared = short side squared + other short side squared

$H^2 = a^2 + b^2$

When finding a short side the formula rearranges to give:

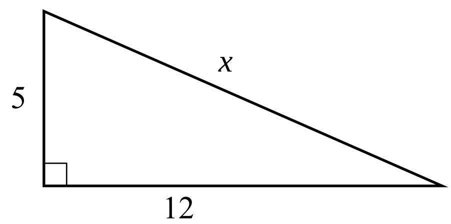
$a^2 = H^2 - b^2$ or $b^2 = H^2 - a^2$

It does not matter which short side is designated “a” and which is “b” for Pythagoras.

It is extremely helpful to set up a layout and routine which you use every time you do a Pythagoras question, as that helps avoid mistakes. The process cannot be short cut. Each side must be squared and added/subtracted, then square rooted to get the answer.

Finding a long side requires a bigger number than the other sides, so you need to add the squares. Finding a short side means that the number must be smaller than the long side, so you need to subtract the squares.

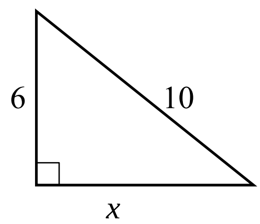
Until you are very comfortable, actually writing out the formula at the start of every exercise is worthwhile. It helps you learn it better, and reminds you whether you are adding or subtracting.



Long side: $H^2 = a^2 + b^2$

$x^2 = 12^2 + 5^2 = 169$

$x = \sqrt{169} = 13$



Short side: $a^2 = H^2 - b^2$

$x^2 = 10^2 - 6^2 = 64$

$x = \sqrt{64} = 8$

A common mistake is to forget to take the square root after adding/subtracting the squares. This can be avoided by (1) always doing your working in the same routine, so it becomes automatic, and (2) always checking your answer to see if it makes sense. A ridiculously large answer suggests that you have forgotten to take the square root.

If you get a negative answer and try to square root it, you will get a Maths error. You always take the short side squared from the Hypotenuse squared ($a^2 = H^2 - b^2$ or $b^2 = H^2 - a^2$).

Pythagoras Theorem is often seen as $a^2 + b^2 = c^2$, but that tends to hide which side is the long side and make it easier to get wrong. It is also confusing in the 3-dimensional case.

Finding a Side Length with Sin/Cos/Tan

After sorting out the 90° triangle you wish to solve and labelling it, you will be left with a side you need to find and a side length you are given.

You select your formula from $\overset{O}{S\quad H}$ $\overset{A}{C\quad H}$ $\overset{O}{T\quad A}$ based on those two sides.
where: H = Hypotenuse, A = Adjacent, O = Opposite, S = Sine, C = Cosine, T = Tangent

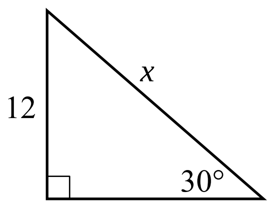
With the selected formula, cover the side being found and that gives the relationship needed.
It is a division if the uncovered letters are on different levels:

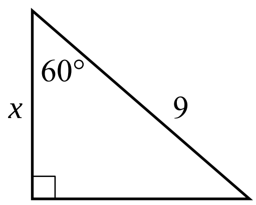
$H = \frac{O}{S}$ $\qquad$ $H = \frac{A}{C}$ $\qquad$ $A = \frac{O}{T}$

It is a multiplication if the uncovered letters are side by side:

$O = S \times H$ $\qquad$ $A = C \times H$ $\qquad$ $O = T \times A$

It is useful to write out the part of SOH CAH TOA being used every time, in order to prevent confusion about what number goes where. Setting up a regular routine and layout is a useful way of ensuring that you make less mistakes and can remember the process later.





We need the H (x) and have the O (12)

That means we use $\overset{O}{S\quad H}$

$H = \frac{O}{S} = \frac{12}{\text{Sin } 30} = 24$

We need the A (x) and have the H (9)

That means we use $\overset{A}{C\quad H}$

H = C × H = Cos 60° × 9 = 4.5

In SOH CAH TOA the "S" is the Sine (SIN) function, the "C" is the Cosine (COS) function and the "T" is the Tangent (TAN) function. These operate only on the **angles**. It is vital that you do not attempt to take the sin/cos/tan of the side length.

Watch that most modern calculators require brackets to separate the angle portion from any calculation. Above, where we want Cos 60° × 9 we need to type in cos(60) × 9 into the calculator.

Up to Year 11 trigonometry is always done using degrees, but your calculator also operates in radians and gradians. You need to ensure it is set correctly – on most scientific calculators this is indicated by a little "D" at the top of the screen. If it is showing "R" or "G" you will need to change it using the Mode button. Graphics Calculators automatically assume you will work in radians, so they must be changed to degrees every time memory is cleared.

Finding an Angle with Sin/Cos/Tan

After sorting out the 90° triangle you wish to solve and labelling it, you will be left with two sides given and an angle to find.

You select your formula from $\overset{O}{S\quad H}$ $\overset{A}{C\quad H}$ $\overset{O}{T\quad A}$ based on those two sides given.

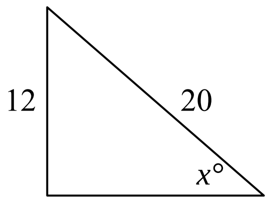
The relationship is given by a division:

$$\text{Sin(angle)} = \frac{O}{H} \qquad \text{Cos(angle)} = \frac{A}{H} \qquad \text{Tan(angle)} = \frac{O}{A}$$

But we need the angle, not the trig(angle), so we move the function across the equals sign and do the opposite:

$$\text{angle} = \sin^{-1}\left(\frac{O}{H}\right) \qquad \text{angle} = \cos^{-1}\left(\frac{A}{H}\right) \qquad \text{angle} = \tan^{-1}\left(\frac{O}{A}\right)$$

It is useful to write out the part of SOH CAH TOA being used every time. Doing this as part of a set routine and layout reduces mistakes and helps make the process automatic. When finding an angle it pays to write out that you are going to use the **inverse** form of sin, cos or tan.

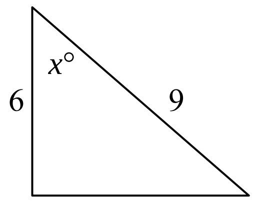


We have the H (20) and O (12)

That means we use $\overset{O}{S\quad H}$

$\sin(x) = \frac{O}{H}$

$x = \sin^{-1}\left(\frac{12}{20}\right) = 36.87°$



We have the H (9) and A (6)

That means we use $\overset{A}{C\quad H}$

$\cos(x) = \frac{A}{H}$

$x = \cos^{-1}(6 \div 9) = 48.19°$

The inverse functions, i.e. $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$, are found on calculators by using the shift key before the normal sin, cos or tan key. You **cannot** type "cos–1" directly into the calculator. If the division is done as one step with the $\cos^{-1}$ etc, it needs to be bracketed on the calculator – so we need to type in shift-sin(12÷20) so that the entire division is acted on by $\sin^{-1}$.

It is vital to get your fraction the right way up and always writing out the formula helps ensure this.

Often Greek letters are used to represent the unknown angles. Generally this is θ (theta), but also α (alpha), β (beta), γ (gamma) and the rest. It's just a letter representing an unknown value and you shouldn't get fixated on it being Greek letter. Make it *x* if it bothers you too much.