Homework 20

Solve:

Simplify

1. $\sqrt{t^3} \times \sqrt{t^3}$

2. $(3x + k)(3x - k)$

3. $\sqrt{16t^6}$

4. $\sqrt{0.01t^3}$

5. $\dfrac{1 - x}{x - 1}$

6. $\dfrac{x^2 + 2x + 1}{x^2 - 1}$

7. $\dfrac{2x - x^2}{x - 2}$

Solve:

8. $p^6 \times p^n = p^2$

9. $(x + 5)(4x + 3) = 0$

10. $25x - 15 = 0$

11. $(\frac{10}{x})^3 = 27$

12. $x^3 = 80x + 2x^2$

13. $(2x + 5)(9 - x) = 0$

14. Solve for n and k:

$(x + 2)(x + n)$

$= x^2 + 7x + k$

Make r the subject:

15. $A = \pi r^2$

16. $\dfrac{1}{r} + 2 = \dfrac{3a}{7}$

17. $y = \dfrac{3a}{r + 5}$

18. $y = \dfrac{2r^3}{5}$

19. $5x^3 = \sqrt{r}$

20. $k = \sin(r)$

21. $k = 2r + \pi r$

Proofs

22. Prove that the sum of any four consecutive numbers added is even.

23. If $t_n = 4n + n^2$ show that the difference from the nth to next terms is $2n + 5$

24. Show that h is never negative if $h = x^2 - 8x + 17$

Answers: Homework 20

Simplify

1. $\sqrt{t^3} \times \sqrt{t^3}$

   $= t^3$

2. $(3x + k)(3x - k)$

   $= 9x^2 - 3kx + 3kx - k^2$

   $= 9x^2 - k^2$

3. $\sqrt{16t^6}$

   $= 4t^3$

4. $\sqrt{0.01t^3}$

   $= 0.1t^{1.5}$

5. $\dfrac{1 - x}{x - 1}$

   $= \dfrac{-(x-1)}{x-1}$

   $= -1$

6. $\dfrac{x^2 + 2x + 1}{x^2 - 1}$

   $= \dfrac{(x+1)(x+1)}{(x+1)(x-1)}$

   $= \dfrac{x+1}{x-1}$

7. $\dfrac{2x - x^2}{x - 2}$

   $= \dfrac{-x(x-2)}{x-2}$

   $= -x$

Solve:

8. $p^6 \times p^n = p^2$

   $6 + n = 2$

   $n = -4$

9. $(x + 5)(4x + 3) = 0$

   $x = -5$ or $x = {}^{-3}/4$

10. $25x - 15 = 0$

    $5(5x - 3) = 0$

    $5 = 0$ or $5x - 3 = 0$

    $x = {}^3/5$

11. $(\frac{10}{x})^3 = 27$

    ${}^{10}/x = 3$

    $x = {}^{10}/3$

12. $x^3 = 80x + 2x^2$

    $x^3 - 2x^2 - 80x = 0$

    $x\,(x^2 - 2x - 80) = 0$

    $x\,(x + 8)(x - 10) = 0$

    $x = 0$, $x = -8$ or $x = 10$

13. $(2x + 5)(9 - x) = 0$

    $x = -2.5$ or $x = 9$

14. Solve for n and k:

    $(x + 2)(x + n)$

    $= x^2 + 7x + k$

    $2 + n = 7 \Rightarrow n = 5$

    $k = 2 \times n = 10$

Make r the subject:

15. $A = \pi r^2$

    ${}^A/\pi = r^2$

    $r = \sqrt{\dfrac{A}{\pi}}$

16. $\dfrac{1}{r} + 2 = \dfrac{3a}{7}$

    $\dfrac{1}{r} = \dfrac{3a}{7} - \dfrac{14}{7} = \dfrac{3a - 14}{7}$

    $r = \dfrac{7}{3a - 14}$

17. $y = \dfrac{3a}{r + 5}$

    $y\,(r + 5) = 3a$

    $r = \dfrac{3a}{y} - 5$

18. $y = \dfrac{2r^3}{5}$

    $\dfrac{5y}{2} = r^3$

    $r = \sqrt[3]{\dfrac{5y}{2}}$

19. $5x^3 = \sqrt{r}$

    $r = 25x^6$

20. $k = \sin(\ r\ )$

    $r = \sin^{-1}(\ k\ )$

21. $k = 2r + \pi r$

    $k = r\,(2 = \pi)$

    $r = \dfrac{k}{2 + \pi}$

Proofs

22. Prove that the sum of any four consecutive numbers added is even.

    Let x be the first of our numbers. The sum is then $x + x + 1 + x + 2 + x + 3 = 4x + 6$

    $4x$ is even and 6 is even, and two evens added are even, so the sum is even.

23. If $t_n = 4n + n^2$ show that the difference from the nth to next terms is $2n + 5$

    $t_{n+1} - t_n = 4(n + 1) + (n + 1)^2 - (4n + n^2) = 4n + 4 + n^2 + 2n + 1 - 4n - n^2 = 2n + 5$

24. Show that h is never negative if $h = x^2 - 8x + 17 = (x - 4)^2 + 1$, so the min is at $x = 4$.

    So the minimum value of $h = (4 - 4)^2 + 1 = 1$, and all other values will be larger.