Homework #21

Solve: (note: you **must** use equations)

1.   Bill has five boxes of pens and 5 single pens. He has 29 pens more than Al, who has three complete boxes of pens. How many pens are in a box?

2.   One side of a rectangle is 5 cm longer than the other and the perimeter is 59 cm. What size is the rectangle?

3.   I think of a number, add five and then square that. The result is 81. What was my number?

4.   Find numbers that when squared are 56 more than the starting number.

5.   What two numbers add to give 8 but have a difference of 11?

6.   Which two consecutive numbers when multiplied together give 90?

7.   Bill earns half of what Emma does. If Bill earned $15,000 more he would earn three-quarters of what Emma does. How much does Emma earn?

8.   Train carriages seat either 24 people or 30 people. If seven carriages can seat a total of 180, how many are there of each type of carriage?

9.   Bob gets a pay rise of $2.50 an hour. That increases his pay by more than a third. What was his original pay?

10.   Three red blocks weight the same as two blue blocks exactly. If five red blocks are heavier than three blue blocks and 250 grams, how much does a red block weigh?

11.   If increasing a circle's radius by six cm means that its area increases to sixteen times what it was before, what was the original radius of the circle?

12.   Which two numbers differ by 10 and have a difference of their squares of 140?

Answers Homework #21

Note that all solutions **must** start from an **algebraic equation**. Merely showing that a solution works is not sufficient. If there are two solutions, **both** must be given.

1. Bill has five boxes of pens and 5 single pens. He has 29 pens more than Al, who has three complete boxes of pens. How many pens are in a box?

    $5b + 5 = 3b + 29$ $2b = 24$ $b = 12$ Each box has 12 pens

2. One side of a rectangle is 5 cm longer than the other and the perimeter is 59 cm. What size is the rectangle?

    $x + x + (x + 5) + (x + 5) = 59$ $4x + 10 = 59$ $x = 49/4 = 12.25$

    rectangle is 12.25 by 17.25

3. I think of a number, add five and then square that. The result is 81. What was my number?

    $(x + 5)^2 = 81$ $x^2 + 10x + 25 = 81$ $x^2 + 10x - 56 = 0$

    $(x + 14)(x - 4) = 0$ $x = 4$ or -14 The number was 4 or –14

4. Find a number that when squared is 56 more than the starting number.

    $x^2 = x + 56$ $x^2 - x - 56 = 0$ $(x + 7)(x - 8) = 0$

    $x = -7$ or 8   the numbers are –7 or 8

5. What two numbers add to give 8 but have a difference of 11?

    $x + y = 8$ and $x - y = 11$ replacing y gives $x + (11 + x) = 8$

    $4x + 11 = 8$ $x = -1.5$ the numbers are –1.5 and 9.5

6. Which two consecutive numbers when multiplied together give 90?

    $x (x + 1) = 90$ $x^2 + x = 90$ $x^2 + x - 90 = 0$ $(x - 9)(x + 10) = 0$

    $x = 9$ or -10 numbers are 9 and 10 or -10 and -9

7. Bill earns half of what Emma does. If Bill earned $15,000 more he would earn three-quarters of what Emma does. How much does Emma earn?

    $B = 0.5 E$ and $B + 15000 = 0.75 E$ so sub out B gets $0.5 E + 15000 = 0.75 E$

    $15000 = 0.25 E$ $E = 60,000$ Emma earns $60,000

8. Train carriages seat either 24 people or 30 people. If seven carriages can seat a total of 180, how many are there of each?

    $a + b = 7$ $24a + 30b = 180$ so $24a + 30(7 - a) = 180$

    $24 a + 210 - 30a = 180$ $30 = 6a$ Five carriages of 24 and two of 30

9. Bob gets a pay rise of $2.50 an hour. That increases his pay by more than a third. What was his original pay?

    $2.5 > \frac{1}{3} x$ $x < 7.5$ It was **less than** $7.50 an hour

10. Three red blocks weight the same as two blue blocks exactly. If five red blocks are heavier than three blue blocks and 250 grams, how much does a red block weigh?

    $3R = 2B$ and $5R > 3B + 250$ so B = 1.5R and so $5R > 3(1.5R) + 250$

    $5R > 4.5R + 250$ $0.5R > 250$ The Red blocks weigh **more** than 500 g

11. If increasing a circle's radius by six cm means that its area increases to sixteen times what it was before, what was the original radius of the circle?

    $\pi (r + 6)^2 = 16(\pi r^2)$ $(r + 6)^2 = 16r^2$ $r^2 + 12r + 36 = 16r^2$

    $0 = 15r^2 - 12r - 36$ $0 = 5r^2 - 4r - 12$ $0 = (5r + 6)(r - 2)$

    $r = 2$ or $-6/5$, but negatives make no sense   radius was 2 cm

12. Which two numbers differ by 10 and have a difference of their squares of 140?

    $a - b = 10$ and $a^2 - b^2 = 140$ $a^2 - (10 - a)^2 = 140$ $a^2 - 100 + 20a - a^2 = 140$

    $20a - 100 = 140$ $20a = 240$ $a = 12$ numbers are 12 and 2