Year 11 Patterns Notes

Linear Patterns

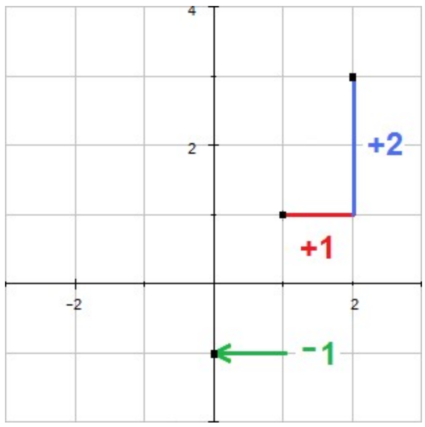
Linear patterns are ones where the difference between one term and the next is constant.

They are "linear"because if plotted they give a line.

They have equations of the form $y = m\,x + c$, just like lines.

x	y
0	−1
1	1
2	3
3	5
4	7

+2
+2
+2



The rule for this pattern is: $y = 2x - 1$.

- The pattern is increasing by two each time, so $m = 2$. Graphically, we get a gradient of 2.
- When $x = 0$, we have $y = -1$, so the c value is -1. Graphically, we get a y-intercept of -1

Generally the pattern given will not start with the zero term, but with the 1st term ($x = 1$). In this case we calculate the zero term as our c. We **never** use the 1st term if it is not $x = 0$.

x	1	2	3	4	5
y	5	8	11	14	17

The rule for this pattern is: $y = 3x + 2$.

- The pattern is increasing by three each time, so $m = 3$.
- When $x = 0$, we would get $y = 5 - 3 = 2$, so the c value is 2.

Patterns will not usually have the variables x and y, but instead variable letters which make sense in the context of the situation. Those variables should used, not x and y.

n	1	2	3	4	5
p	5	15	25	35	45

The rule for this pattern is: $p = 10n - 5$.

- The pattern is increasing by ten each time, so $m = 10$.
- When $n = 0$, we would get $p = 5 - 10 = -5$, so the c value is -5.

The most common mistake is not setting the constant c to what y is when the x value is 0.

Quadratic Patterns

Quadratic patterns are ones where the difference of the differences between terms is constant.

They have equations of the form $y = a\,x^2 + b\,x + c$.

If plotted they give a parabola, as quadratics do.

Method

1) We take the difference of the differences and **halve** it to find the a value.
2) We calculate the c value from the $x = 0$ value, as with lines.
3) After we take away the ax^2 and c parts, then what is left over is the bx term.

x	y
0	3
1	7
2	13
3	21

+4
+6
+8

+2
+2

The rule for this pattern is: $y = x^2 + 3x + 3$.

- The difference of the differences is 2, which we halve to get $a = 1$.
- When $x = 0$, we have $y = 3$, so the c value is 3.
- If we take $1x^2 + 3$ off, the remaining amounts go 0, 3, 6, 9 which is $3x$, and so $b = 3$.

As with lines, generally the pattern will not start with the zero term, but with $x = 1$. In this case we calculate backwards to our $x = 0$ term to find the c value.

x	1	2	3	4	5
y	5	9	17	29	45

The rule for this pattern is: $y = 2x^2 - 2x + 5$.

- The differences are 4, 8, 12, 16, so the difference of differences is 4. Halving gives $a = 2$.
- The differences are 4, 8, 12, 16 so the difference to $x = 0$ is 0, so the c value is 5.
- If we take $2x^2 + 5$ off the remaining amounts go –2, –4, –6, –8 which is $-2x$, and so $b = -2$.

As with linear patterns we need to use the variable letters supplied.

n	1	2	3	4	5
p	100	100	99	97	94

The rule for this pattern is: $p = -0.5n^2 + 1.5n + 100$.

- The differences are 0, $^-1$, $^-2$, $^-3$ so the difference of differences is $^-1$. Halving gives $a = -\frac{1}{2}$.
- The differences are 0, $^-1$, $^-2$, $^-3$ so the difference to $x = 0$ is 1, so the c value is 99.
- If we take $^-0.5n^2 + 99$ off the remaining amounts go 1.5, 3, 4.5, 6 which is $1.5n$, and so $b = 1.5$ (you have to be very careful with the subtracting negatives in cases like these).

Exponential Patterns

Exponential patterns are ones where the terms are increased by a common multiplication.

They have equations of the form $y = c \times b^x$.

They are called exponential because the x is an exponent.

Method

1) The multiplication value between terms is the base, b.
2) We calculate the c value from the $x = 0$ value, although this time it is multiplied, not added.

x	y
0	3
1	6
2	12
3	24

$\times 2$

$\times 2$

$\times 2$

The rule for this pattern is: $y = 3 \times 2^x$.

- Each term is twice the previous one, so $b = 2$.
- When $x = 0$, we have $y = 3$, so the c value is 3.

As with lines and quadratics, generally the zero term will not be given, but the first term given will be the one for $x = 1$. In this case we calculate backwards to our $x = 0$ term to find the c value.

x	1	2	3	4
y	2	10	50	250

The rule for this pattern is: $y = 0.4 \times 5^x$.

- Each term is five times the previous one, so $b = 5$.
- When $x = 0$, we have a fifth of the previous value, so $y = 2 \div 5$, and the c value is 0.4.

If the pattern is decreasing we don't change the process. However the base, b, will be less than one.

x	1	2	3	4
y	80	40	20	10

The rule for this pattern is: $y = 160 \times 0.5^x$.

- Each term is half the previous one, so $b = 0.5$.
- When $x = 0$, we have a twice the previous value, so $y = 80 \times 5$, and the c value is 160.

There are multiple ways of writing many exponential equations. In particular the answers are often given with changes to the x value, rather than a multiplication by a constant c out in front. For example, $y = 2 \times 2^x$ can be written as $y = 2^{x+1}$. There is no advantage to writing the answers with the shift to the x (and it makes for much harder equations to work with and solve). I suggest you stick with what is easiest for you.

Types of Questions

If the question asks for a "rule" or "relationship" then it is simply asking for the equation.

Do not forget to write as an **equation**, with the "$\boldsymbol{y} =$" or equivalent.

If they leave a gap opposite a variable, they want an expression from the rule for that variable:

x	1	2	3	4	n
y	16	20	24	28	

The rule here is: $y = 4x + 12$. So for $x = n$ then $y = 4n + 12$. And we put "$4n + 12$" in the space.

Generally the best way to solve a question is to find the relationship (equation) and then use that.

Numerical and guess and check methods often work, but you risk losing a grade in the marking if your method is considered to be at a lower level of thinking than using algebra.

a	1	2	3	4	...	25
b	200	195	190	185	...	?

The rule for this pattern is: $b = {}^{-}5a + 205$. (**not** + 200, because that's not the value at $a = 0$.)

So when $a = 25$, then $b = {}^{-}5 \times 25 + 205 = 80$.

We can solve this problem without an equation, because we know that from $a = 4$ to $a = 25$ that the b value must drop by $25 – 4 = 21$ lots of 5 less than 185. That gives $185 – 21 \times 5 = 80$.

t	1	2	3	4	...	x
A	6	6.5	7	7.5	...	80

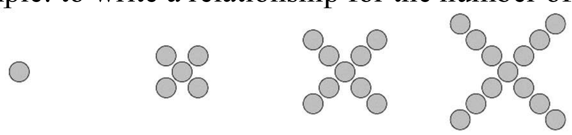
The rule for this pattern is: $A = 0.5t + 5.5$.

So when $A = 80$, then $80 = 0.5t + 5.5$. Solving that equation gives $x = 149$.

We could also solve this by knowing that from 7.5 to 80 is 72.5, which is 145 lots of 0.5. That means $x = 4 + 145 = 149$, even without being able to write the rule.

If a pattern is given as diagrams, then convert it to a numerical table first.

Example: to write a relationship for the number of dots to each place in the following pattern:



Convert to the table below, where d is the number of dots, and p the place in the pattern

p	1	2	3	4
d	1	5	9	13

and get the linear equation as usual: $d = 4p – 3$

Sequences

Sequences are patterns given only as the "y" part.

For example a sequence might be written: 3, 7, 11, 14, 17 …

Each number in the sequence is a term, and are called by their position. So the first term (t_1) above is 3, the second term (t_2) above is 7, etc

We deal with them like the equivalent patterns, except that formally the y value is replaced by t_n (for the nth term) and instead of x we use n. The rule for 3, 7, 11, 14, 17 … is therefore $t_n = 4n - 1$.

Generally it pays to convert sequences to a table.

The sequence 3, 7, 13, 21, 31 … converts to

n	1	2	3	4	5
t_n	3	7	13	21	31

Where the differences are 4, 6, 8, 10 …, increasing by 2, and the 0 term would be 3 – 2 = 1.

And the rule is: $t_n = n^2 + n + 1$.

Graphing

Patterns and sequences are graphed as the equivalent line, parabola or exponential curve.

There are two significant differences however:

1) there are no values between terms so the x values, or equivalent, are **not** connected by lines. Instead we plot separated dots.

2) almost always the pattern makes not sense for x values that are negative – and usually the $x = 0$ value also has no meaning. They should not be plotted in those situations. (Note even though we need to calculate what the $x = 0$ value would be for figuring out the c value in the equation, we do not plot it).