L1 Algebra Trial #4

Q1. a) For x any positive number, which expression is always bigger: $\frac{2x^3}{6x^2}$ or $\frac{x}{2}$?

b) Show that $(2 - x)(4 - x)$ can also be written as $(3 - x)^2 - 1$

c) Show that $\frac{25x^2 - 10x}{25x^2 - 4}$ can also be written as $\frac{5x}{5x + 2}$

d) If $\frac{\sqrt{81x^8}}{ax^n} = 1$ what are the values of a and n ?

e) Make k the subject of the equation: $x = \frac{7}{\sqrt{k + 5}}$

f) The pattern 5, 10, 17, 26, … is given by the rule $t_n = (n + 1)^2 + 1$. Show that the difference between one term and the next is given by: difference = 2n + 3

Q2. a) Show that one of the x-intercepts of $y = 5x^2 - 6x + 1$ is five times the other

b) The height of a bridge is given by the formula, $h = 2x - \frac{1}{4}x^2 + 4$, where x measures the distance out from one end. How high is the bridge at 6m out?

c) For bridge, $h = 2x - \frac{1}{4}x^2 + 4$, where is the other point the same height as in b)?

d) Find the value(s) of x for which $\frac{x + 6}{x} = 3x - 6$

e) $4x^2 - 100x + 625 = 0$ has only one solution, at $x = 25$. Explain what having only one solution means in terms of graphing such a relationship.

f) If $ab^2 = 90$ and $ab = 15$, what is a?

Q3. a) How can $(x + 4)^2 - 2(x + 5)$ be written in the fewest possible terms?

b) Show that $x(x + 5) - 5(x - 3)$ is always a positive number

c) Where does the line $y = 2x - 7$ cross the line $y = 13 - 3x$?

d) Find the smallest integer x where $\frac{x + 4}{3}$ is less than $x + 2$.

e) Define k so that both these following statements are true:
k plus 3 is greater than 8 and 9 minus k is greater than zero.

f) Find a number such that a third of it added to a fifth of it equals 8.

L1 Algebra Trial #4 : Answers

Colours indicate the **approximate** point when Achieved, Merit and Excellence are reached.

Q1. a) $\frac{2x^3}{6x^2} = \frac{2 \times 1 \times x \times x \times x}{2 \times 3 \times x \times x} = \frac{x}{3}$ which is smaller than $\frac{x}{2}$ (as $\frac{1}{3} < \frac{1}{2}$)

b) $(2 - x)(4 - x) = 8 - 2x - 4x + x^2 = x^2 - 6x + 8$ which is the same as

$(3 - x)^2 - 1 = (3 - x)(3 - x) - 1 = 9 - 3x - 3x + x^2 - 1 = x^2 - 6x + 8$

c) $\frac{25x^2 - 10x}{25x^2 - 4} = \frac{5x(5x - 2)}{(5x + 2)(5x - 2)} = \frac{5x\cancel{(5x - 2)}}{(5x + 2)\cancel{(5x - 2)}} = \frac{5x}{5x + 2}$

d) $\sqrt{81x^8} = ax^n$ so $\sqrt{81} \times \sqrt{x^8} = ax^n$ $9x^4 = ax^n$ so $a = 9$ and $n = 4$

e) $x = \frac{7}{\sqrt{k + 5}}$ $\sqrt{k + 5} = \frac{7}{x}$ $k + 5 = \frac{49}{x^2}$ $k = \frac{49}{x^2} - 5$ or $k = \frac{49 - 5x^2}{x^2}$

f) diff = $t_{n+1} - t_n$ = $[(n + 1 + 1)^2 + 1] - [(n + 1)^2 + 1]$

$= (n^2 + 4n + 4 + 1) - (n^2 + 2n + 1 + 1)$ diff = $2n + 3$

Q2. a) $y = 5x^2 - 6x - 1 = (5x - 1)(x - 1)$ so intercepts are $\frac{1}{5}$ and 1, which is 5× as much

b) $h = 2(6) - \frac{1}{4}(6)^2 + 4 = 12 - \frac{36}{4} + 4 = 7$

c) $h = 2x - \frac{1}{4}x^2 + 4 = 7$ $0 = \frac{1}{4}x^2 - 2x + 3$ multiply by 4, $0 = x^2 - 8x + 12$

$0 = (x - 2)(x - 6)$ so at $x = 2$ and $x = 6$, so second point is 2m out

d) Solve: $\frac{x + 6}{x} = 3x - 6$ $x + 6 = x(3x - 6)$ $x + 6 = 3x^2 - 6x$

$0 = 3x^2 - 7x - 6 = (3x + 2)(x - 3)$ $x = 3$ or $\frac{-2}{3}$

e) It is a parabola which turns just as it touches the x-axis – in this case at (25, 0).

f) $b = \frac{ab^2}{ab} = \frac{90}{15} = 6.$ As $ab = 15$, $a \times 6 = 15$, so $a = \frac{15}{6}$ $a = 2.5$

Q3. a) $(x + 4)(x + 4) - 2(x + 5) = x^2 + 8x + 16 - 2x - 10 = x^2 + 6x + 6$ (any order)

b) $x(x + 5) - 5(x - 3) = x^2 + 5x - 5x + 15 = x^2 + 15$. As x^2 is always at least 0, then $x^2 + 15$ is always more than zero i.e. it is positive.

c) $y = 2x - 7$ crosses $y = 13 - 3x$ when $2x - 7 = 13 - 3x$ $5x = 20$ $x = 4$

at $x = 4$, $y = 2 \times 4 - 7 = 1$, so the point (4, 1)

d) Solve: $\frac{x + 4}{3} < x + 2$ $x + 4 < 3(x + 2)$

$x + 4 < 3x + 6$ $^{-}2 < 2x$ $x > {}^{-}1$, so smallest integer is zero

e) $k + 3 > 8$ so $k > 5$ $9 - k > 0$ $^{-}k > {}^{-}9$ so $k < 9$

$5 < k < 9$ or in words, such as "k is any number more than 5 but less than 9"

f) $\frac{x}{3} + \frac{x}{5} = 8$ $\frac{5x}{15} + \frac{3x}{15} = 8$ $8x = 15 \times 8$ $x = 15$

the number is 15 (must solve using equations)