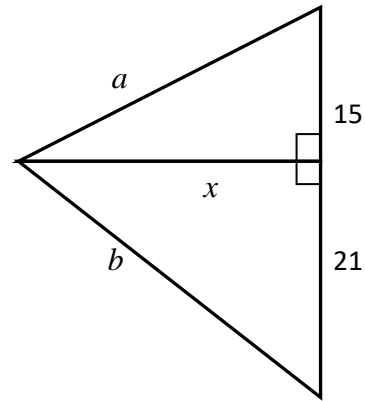


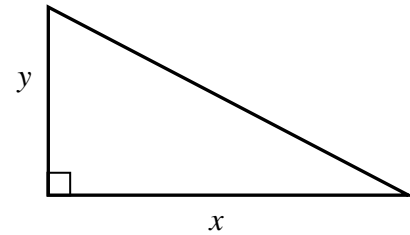
Year 12 Algebra Excellence #1

1. Find x so that b is 4 more than a .



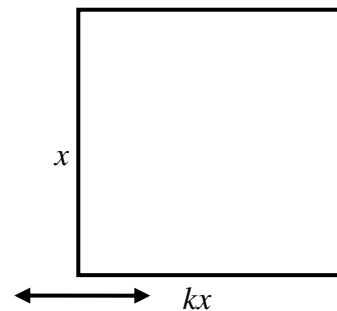
2. Factorise: $3^{x+3} - 9^x$

3. Write an expression for x in terms of y if the sum of x and y is equal to the triangle's area.



4. Solve: $x + \sqrt{x} = 156$

5. A square has side length x .
It is changed to a rectangle by multiplying one of the sides by a multiple, k .
Find k in terms of x , if the area of the new rectangle is the same number as the perimeter.
Simplify your answer fully.



6. Find k so that $y = 4x^2 + kx + 15$ has two x -intercepts that are exactly 1 apart.

7. Make x the subject of the equation: $4^{x+1} = k^x$

8. Find both the solutions to: $a^2 + ab = 2b^2$ for a in terms of b

Answers: Year 12 Algebra Excellence #1

1. Find x so that b is 4 more than a .

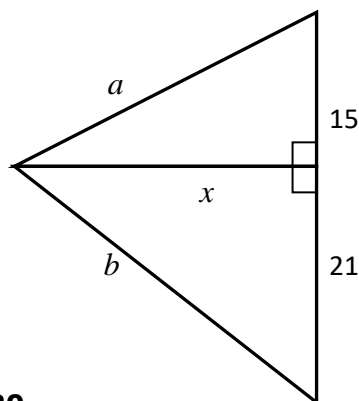
$$x^2 + 15^2 = a^2 \text{ and } x^2 + 21^2 = (a + 4)^2$$

$$x^2 = a^2 - 15^2 \text{ and } x^2 = (a + 4)^2 - 21^2$$

$$a^2 - 225 = a^2 + 8a + 16 - 441$$

$$a = 25$$

$$x^2 + 15^2 = a^2 \text{ so } x^2 + 225 = 625 \quad \mathbf{x = 20}$$



2. Factorise $3^{x+3} - 9^x$

$$= 3^{x+3} - 3^{2x}$$

$$= 3^x(3^3 - 3^x)$$

$$= \mathbf{3^x(27 - 3^x)}$$

3. Write an expression for x in terms of y

if the sum of x and y is equal to the triangle's area.

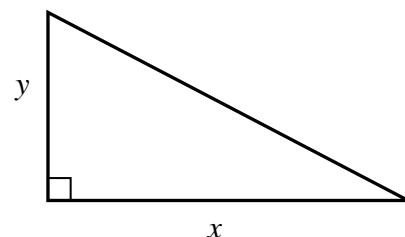
$$x + y = \frac{1}{2}xy$$

$$x - \frac{1}{2}xy = y$$

$$x(1 - \frac{1}{2}y) = y$$

$$x = \frac{y}{1 - \frac{1}{2}y}$$

$$\mathbf{x = \frac{2y}{2-y}}$$



4. Solve $x + \sqrt{x} = 156$

$$\Rightarrow x + \sqrt{x} - 156 = 0$$

$$\Rightarrow (\sqrt{x} + 13)(\sqrt{x} - 12) = 0$$

$$\Rightarrow \sqrt{x} = -13 \text{ or } \sqrt{x} = 12$$

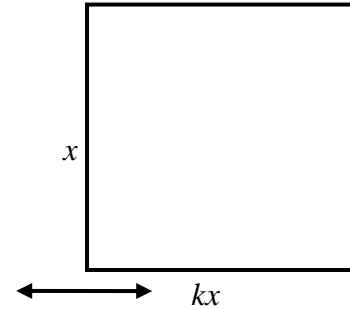
first is not possible, second when $x = \mathbf{144}$

5. A square has side length x .

It is changed to a rectangle by multiplying one of the sides by a multiple, k .

Find k in terms of x , if the area of the new rectangle is the same number as the perimeter.

Simplify your answer fully.



$$kx^2 = 2x + 2kx \Rightarrow k(x^2 - 2x) = 2x$$

$$k = \frac{2x}{x^2 - 2x} \Rightarrow k = \frac{2}{x - 2}$$

6. Find k so that $y = 4x^2 + kx + 15$ has two x -intercepts that are exactly 1 apart.

x -intercepts on graph are roots of the factorised equation.

$$\Rightarrow 4x^2 + kx + 15 = 4(x - r)(x - r + 1) \quad r \text{ is one root, so } r + 1 \text{ is the other}$$

$$\Rightarrow 4x^2 + kx + 15 = 4x^2 + (8r + 4)x + (4r^2 + 4r)$$

$$\text{coefficients must match} \Rightarrow 4r^2 + 4r = 15 \quad \Rightarrow \quad r = 1.5 \text{ or } -2.5$$

$$\text{again coefficients must match} \Rightarrow k = 8r + 4 \quad \Rightarrow \quad k = \mathbf{-16 \text{ or } 16}$$

7. Make x the subject of the equation: $4^{x+1} = k^x$

$$\Rightarrow (x + 1) \log 4 = x \log (k) \quad \text{or} \quad \Rightarrow x + 1 = x \log_4 (k)$$

$$\Rightarrow x \log 4 - x \log k = -\log 4 \quad \Rightarrow x - x \log_4 (k) = -1$$

$$\Rightarrow x (\log 4 - \log k) = -\log 4 \quad \Rightarrow x = \frac{-1}{1 - \log_4 k} = \frac{1}{\log_4 k - 1}$$

$$\Rightarrow x = \frac{-\log 4}{\log 4 - \log k} = \frac{\log 4}{\log k - \log 4} = \frac{\log 4}{\log(\frac{k}{4})} \text{ etc}$$

8. Find both the solutions to: $a^2 + ab = 2b^2$ for a in terms of b

$$a^2 + ab = 2b^2 \quad \Rightarrow a^2 + ab + \frac{1}{4}b^2 = 2b^2 + \frac{1}{4}b^2$$

This is "completing the square" because $(a + \frac{1}{2}b)^2 = a^2 + ab + \frac{1}{4}b^2$

$$\Rightarrow (a + \frac{1}{2}b)^2 = 2.25b^2 \quad \Rightarrow a + 0.5b = \pm\sqrt{2.25}b = \pm 1.5b \quad \text{Note } \pm$$

$$a = -1.5b - 0.5b \text{ or } +1.5b - 0.5b \quad a = \mathbf{-2b \text{ or } b}$$