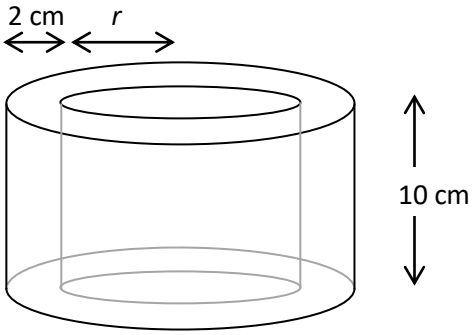
Year 12 Algebra Excellence #2

1. $4x^2 + 56x + c$ has two roots, one which is three more than the other. Find c.

2. Show that $k = 1.2 \times 1.5^{3t}$ rearranges to give $t = \dfrac{\log k - \log 1.2}{\log 3.375}$

3. Solve: $a - 4\sqrt{a} = 5$

4. A hollow cylinder has an edge thickness of 2 cm and a height of 10 cm.
   Find the inner radius, r, in terms of the volume, V.
   Simplify your answer fully.

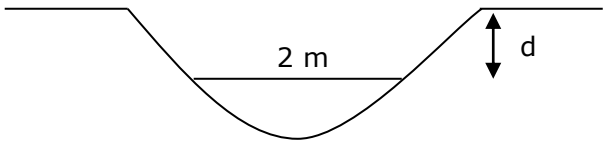


5. Solve: $\dfrac{x^2 + 13x + 30}{x^2 + 9x + 18} = 2$

6. Simplify fully: $\dfrac{y^{2x}}{(2y)^{5x}}$

7. For what values of k does the parabola $y = 4x^2 + kx + k + 2$ not intersect the line $y = 5$?

8. A ditch is parabolic shaped, 3 metres wide, and 1.2 metres deep. A 2 metre plank is placed in it, level to the ground. How deep (d) would that plank be?



Answers: Year 12 Algebra Excellence #2

1. $4x^2 + 56x + c$ has one root three more than the other. Find c.

   $4(x + r)(x + [r + 3]) = 4x^2 + 56x + c$

   $4(x^2 + x[r + 3] + xr + r[r + 3]) = 4x^2 + 56x + c$

   $4x^2 + (8r + 12)\,x + (4r^2 + 12r) = 4x^2 + 56x + c$

   so $8r + 12 = 56$, so $r = 8.5$ put into $c = 4r^2 + 12r$

   $c = 147$

   testing on graphics: $4x^2 + 56x + 187 = 4(x + 5.5)(x + 8.5) = (2x + 11)(2x + 17)$

2. Show that $k = 1.2 \times 1.5^{3t}$ rearranges to give $t = \dfrac{\log k - \log 1.2}{\log 3.375}$

   $k = 1.2 \times 1.5^{3t}$

   $\Rightarrow$ $\log\left(\frac{k}{1.2}\right) = 3t \log 1.5$

   $\Rightarrow$ $3t = \dfrac{\log k - \log 1.2}{\log 1.5}$ as log (a ÷ b) = log a – log b

   $\Rightarrow$ $t = \dfrac{\log k - \log 1.2}{3 \log 1.05}$ but 3 log 1.5 = log (1.5^3)

   $\Rightarrow$ $\boldsymbol{t = \dfrac{\log k - \log 1.2}{\log 3.375}}$

3. Solve: $a - 4\sqrt{a} = 5$

   Let $x = \sqrt{a}$ which means our equation becomes $x^2 - 4x - 5 = 0$

   $\Rightarrow$ $x = 5$ or $^-1$

   $\Rightarrow$ as $x = \sqrt{a}$ then $a = x^2$

   $\Rightarrow$ $a = 25$ or 1. But $(^-1)^2$ loses the sign, and $(1) - 4\sqrt{(1)} = 5$ is not true

   Answer $a = 25$

4. A hollow cylinder has an edge thickness of 2 cm and a height of 10 cm.

   Find the inner radius, r, in terms of the volume, V.

   $V = \pi r_o^2 h - \pi r_i^2 h$ outside volume minus inside volume

   $\Rightarrow$ $V = 10\pi\,((r + 2)^2 - r^2)$ as $r_i = r$ and $r_o = r + 2$ and $h = 10$

   $\Rightarrow$ $V = 40\pi(r + 1)$

   $\Rightarrow$ $r + 1 = V \div 40\pi$

   Answer $r = \dfrac{V}{40\pi} - 1$ or $r = \dfrac{V - 40\pi}{40\pi}$

5. Solve: $\dfrac{x^2 + 13x + 30}{x^2 + 9x + 18} = 2$

$\Rightarrow \quad x^2 + 13x + 30 = 2x^2 + 18x + 36$

$\Rightarrow \quad 0 = x^2 + 5x + 6 \qquad \Rightarrow \quad 0 = (x + 2)(x + 3) \qquad \Rightarrow \quad x = {}^-2$ or $x = {}^-3$

But $x^2 + 9x + 18 = 0$ when $x = {}^-3$, so it gives division by zero

(Students need to always consider that the divisor might be zero in questions)

Answer: $x = {}^-2$

Alternatively: because $\dfrac{x^2 + 13x + 30}{x^2 + 9x + 18} = \dfrac{(x + 10)(x + 3)}{(x + 6)(x + 3)} = \dfrac{x + 10}{x + 6}$ for $x \neq {}^-3$

$\Rightarrow \quad 2 = \dfrac{x + 10}{x + 6} \qquad \Rightarrow \quad 2x + 12 = x + 10 \qquad \Rightarrow \quad x = {}^-2$

(Need to consider $x = {}^-3$ solution from the division step, but it doesn't work)

Answer: $x = {}^-2$

6. Simplify fully: $\dfrac{y^{2x}}{(2y)^{5x}}$

$\dfrac{y^{2x}}{(2y)^{5x}} = \dfrac{y^{2x}}{2^{5x}y^{5x}} = \dfrac{y^{2x}}{2^{5x}y^{3x}\,y^{2x}}$ and $2^{5x} = (2^5)^x = 32^x$

Answer: $\dfrac{\mathbf{1}}{\mathbf{32^x y^{3x}}}$ or $\dfrac{\mathbf{1}}{\mathbf{(32y^3)^x}}$

7. For what values of k does the parabola $y = 4x^2 + kx + k + 2$ not intersect the line $y = 5$?

Intersect when $y = y$ for some x, so $4x^2 + kx + k + 2 = 5$

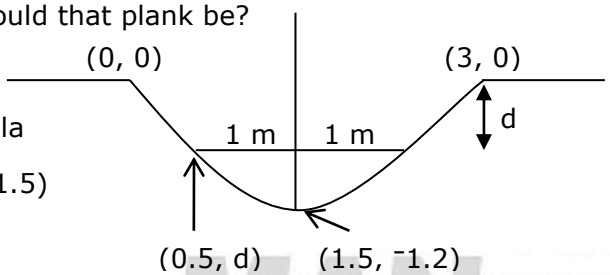
$\Rightarrow \quad 4x^2 + k\,x + (k - 3) = 0$

No intersections when $\Delta < 0$, so $k^2 - 4 \times 4 \times (k - 3) < 0$

$\Rightarrow \quad k^2 - 16k + 48 < 0 \qquad \Rightarrow \quad (k - 4)(k - 12) < 0$

Answer: $4 < k < 12$

8. A ditch is parabolic shaped, 3 metres wide, and 1.2 metres deep. A 2 metre plank is placed in it, level to the ground. How deep (d) would that plank be?



Use co-ordinate system shown on diagram:

$\Rightarrow \quad$ depth = $k\,x\,(3 - x)$ is formula for parabola

Point (1.5, ⁻1.2) on it, so $^-1.2 = k \times 1.5\,(3 - 1.5)$

$\Rightarrow \quad k = 0.5333333\ldots$

Solving for $x = 0.5$ (1 m from the middle at $x = 1.5$) in depth = $0.5333 \times 0.5\,(3 - 0.5)$

Answer $= \dfrac{\mathbf{2}}{\mathbf{3}}$ metres down