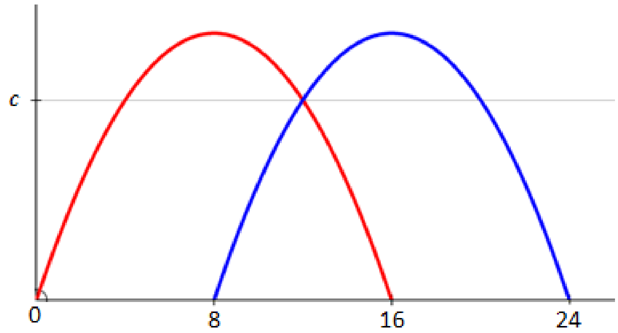
Year 12 Algebra Excellence #3

1. Solve $5x^4 + 20x^2 = 160$.

2. Write x in terms of k if $4^x = 2^{k+2}$.

3. If $\log_b 5 = k$ write $\log_b 0.2$ in terms of k.



4. Calculate the maximum height of the parabolas shown to the right in terms of c, the height at which they intersect.

5. If $\log_b 2 = 0.7737$ and $\log_b 3 = 1.2236$ calculate b **exactly**.

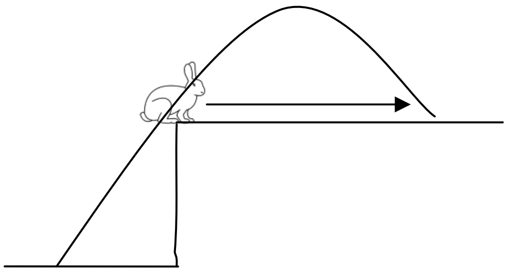
6. Solve for k in terms of x if $4^k = \sqrt{\frac{16^x}{4^x}}$

7. A bank offers interest at 5% per annum, compounding and payable monthly on an initial sum of $1,000.

   Write an equation, using the monthly interest rate, that allows you to calculate the amount of money at the end of each month in terms of t, the time in months.

8. A stone is thrown (in a parabola) so it would hit a rabbit at the edge of a cliff. At the moment the stone is thrown the rabbit runs away at 12 metres per second, The stone reaches a height twice that of the cliff. On the way down hits the rabbit.

   Find the horizontal component of the stone's velocity.



Answers: Year 12 Algebra Excellence #3

1. Solve $5x^4 + 20x^2 = 160$.

   $5(x^4 + 4x^2 - 32) = 0$

   $5(k^2 + 4k - 32) = 0$ where $k = x^2$,

   $(k + 8)(k - 4) = 0$ so $(x^2 + 8)(x^2 - 4) = 0$

   $\Rightarrow x^2 = {}^{-}8$ or $x^2 = 4$

   Answer: $\boldsymbol{x = \pm 2}$

2. Write x in terms of k if $4^x = 2^{k+2}$

   $\Rightarrow \quad (2^2)^x = 2^{k+2}$ since $4 = 2^2$

   $\Rightarrow \quad 2^{2x} = 2^{k+2}$

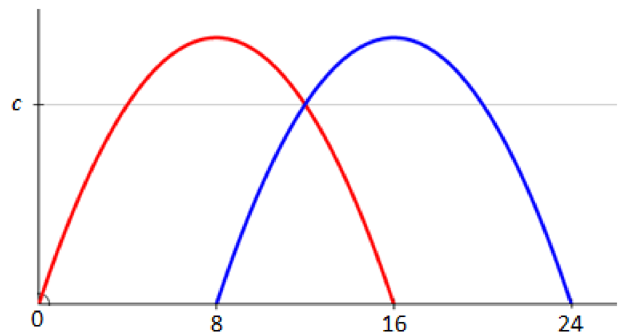
   $\Rightarrow \quad 2x = k + 2$

   Answer: $\boldsymbol{x = \frac{k+2}{2}}$ **or** $\boldsymbol{x = \frac{1}{2}k + 1}$

3. If $\log_b 5 = k$ write $\log_b 0.2$ in terms of k.

   $\log_b 0.2 = \log_b \frac{1}{5} = \log(5^{-1}) = {}^{-}1 \log 5 = {}^{-}k$

   Answer: $\boldsymbol{\log_b 0.2 = {}^{-}k}$



4. Calculate the maximum height of the parabolas shown to the right in terms of c, the height at which they intersect.

   The red has the form $y = k\,x(x - 16)$

   and goes through $(12, c)$ – by taking the midpoint of the blue and red.

   Putting those two together $c = k \times 12 \times (12 - 16)$ so $k = \frac{c}{48}$

   The highest point of the red is at the midpoint, $x = 8$

   Highest $y = \frac{c}{48} \times 8 \times (8 - 16)$

   Answer: $\boldsymbol{y = \frac{4c}{3}}$ **or** $\boldsymbol{= \frac{4}{3}c}$

   Alternatively, use your knowledge that any parabola is of the form $y = k\,x^2$ so that for every doubling of x distance from turning point leads to a quadrupling of y distance.

   From $x = 8$ to $x = 12$ is a height, say k for a x distance of 4.

   From $x = 8$ to $x = 16$ must therefore be a height of $4k$ because it is 8 across.

   The change increase of $3k$ is equal to c, so the full $4k$ to the top must be $= \frac{4}{3}c$.

5. If $\log_b 2 = 0.7737$ and $\log_b 3 = 1.2236$ calculate b **exactly**.

$\log_b 2 + \log_b 3 = 0.7737 + 1.2236$

$\Rightarrow \quad \log_b (2 \times 3) = \log_b 6 = 2$

If $\log_b 6 = 2$ then $b^2 = 6$

Answer: b = √6

Note: if	$\log_b 2 = 0.7737$
then	$2 = b^{0.7737}$
and	$b = \sqrt[0.7737]{2} = 2.4495$
this answer is close but **not exact**	

6. Solve for k in terms of x if $4^k = \sqrt{\frac{16^x}{4^x}}$

$4^k = \sqrt{\frac{16^x}{4^x}} = \sqrt{\frac{(4^2)^x}{4^x}} = \sqrt{\frac{4^{2x}}{4^x}} = \sqrt{4^x} = 4^{0.5x}$ (also $2^x = 4^{0.5x}$)

Answer: $k = ½x$ or $\frac{x}{2}$

Note $\sqrt{x^1} = x^{0.5}$ – the base stays the same

7. A bank offers interest at 5% per annum, compounding and payable monthly on an initial sum of $1,000. Write an equation, using the monthly interest rate, that calculates the amount of money at the end of each month in terms of t, the time in months.

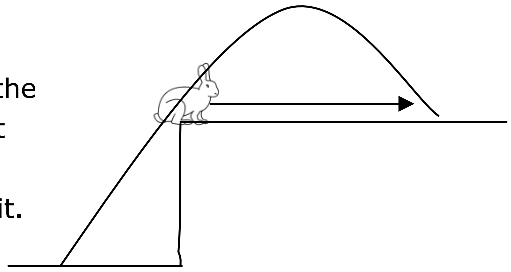
The general form is Balance = 1000 × rate ^{t} , where t is time in months.

The amount per month must be such that $r^{12} = 1.05$

$\Rightarrow r = \sqrt[12]{1.05} = 1.004074$

Answer: Monthly balance = 1000 × 1.004074 ^{t}

(Note this is mathematically the same as Monthly balance = 1000 × 1.05 $^{(t/12)}$)



8. A stone is thrown (in a parabola) so it would hit a rabbit at the edge of a cliff. At the moment the stone is thrown the rabbit runs away at 12 metres per second, The stone reaches a height twice that of the cliff. On the way down hits the rabbit.

Find the horizontal component of the stone's velocity.

Make the rabbit at start = (0,0) and at end (12, 0). The stone's path is $y = k\,x\,(x - 12)$

k can be any negative value, so make it = ⁻1. That makes the top of the path when $x = 6$, so $y_{max} = {}^-1 \times 6 \times (6 - 12) = 36$.

The top of the path is the same as the height of cliff, which must be $y = {}^-36$.

Solving $^-36 = {}^-1\,x\,(x - 12)$ gives $0 = x^2 - 12x - 36$ so $x = {}^-2.485$ and 14.485.

Add the 2.485 on to the 12 metres the rabbit ran. Ran 12 m, so must have taken 1 sec.

Answer: the stone travels 14.485 m s^{-1} horizontally

Alternatively: again make the distance the rabbit runs = 12 m. The general form of a parabola is $y = x^2$, so then y is doubled, x increases by √2. So the distance from the top of the parabola to the bottom of the cliff is √2 × 6. Add the other 6 metres on the other side. That is √2 × 6 + 6 = 14.485 metres. In one second ⇒ **14.485 m s^{-1}**