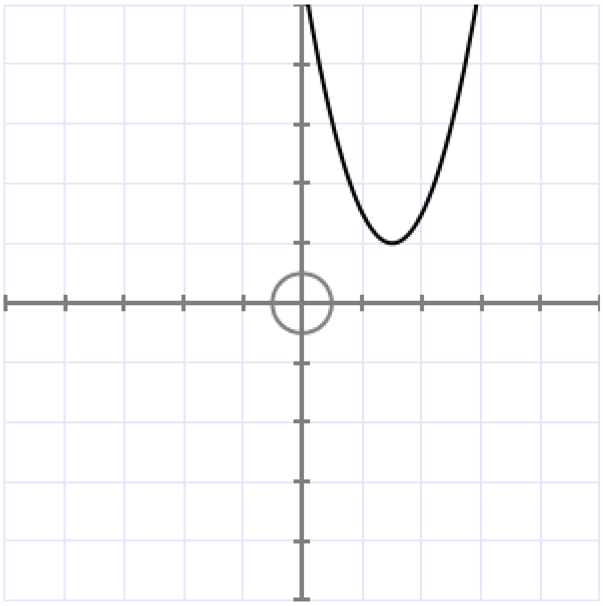
L2 Calculus Practice #1

1. Find the gradient of the curve $y = 6x^2 - 10x + 5$ at the point where $x = 2$.

2. Sketch the derivative function for the parabola shown to the right.



3. The gradient function for a curve is $f'(x) = 5x - 6x^2$.

   The curve passes through the point (2, 3)

   Find the equation of the curve.

4. For the graph of the equation $y = 6.5x^2 - x^3 + 7$ find the x–coordinates of the points on the graph where the gradient is 4.

5. The depth of a dam is given by $d = 30 + 4.2t - 1.2t^2$

   where d is the depth (metres) and t is the time (days).

   Calculate when the maximum depth of the dam.

6. Find the equation of the tangent to the function $p(x) = 3x^2 - 4x + 8$ at $x = 6$.

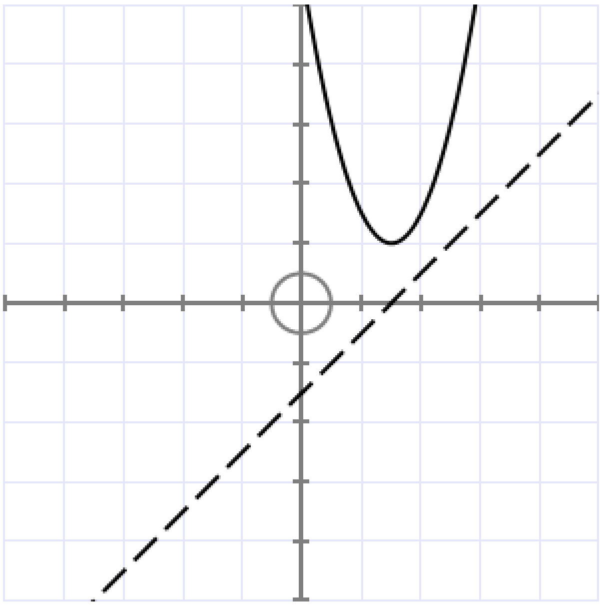
Answers: L2 Calculus Practice #1

1. $y = 6x^2 - 10x + 5$ so $\frac{dy}{dx} = 12x - 10$

   At $x = 2$, $= 12 \times 2 - 10 = 14$

   Gradient = 14

2. **Drawn**



   It must be – a straight line

   – any positive slope

   – with the shown x-intercept

3. $f\,'(x) = 5x - 6x^2$ so $f(x) = 2.5\,x^2 - 2x^3 + C$

   Passes through (2, 3) so $3 = 2.5 \times 2^2 - 2 \times 2^3 + C$. Solving gives C = 9

   Equation is $y = 2.5x^2 - 2x^3 + 9$

4. $y = 6.5x^2 - x^3 + 7$ so $\frac{dy}{dx} = 13x - 3x^2$

   We want when $4 = 13x - 3x^2$ Rearranging gives $3x^2 - 13x + 4 = 0$

   Solve on calculator or factorising to $(3x - 1)(x - 4) = 0$

   Solutions at $x = 4$ and $x = \frac{1}{3}$

5. $d = 30 + 4.2t - 1.2t^2$ so rate of change of depth, $\frac{\mathrm{dd}}{\mathrm{dt}} = 4.2 - 2.4t$

   Maximum when rate = 0 at top of parabola. $0 = 4.2 - 2.4t$, so when $t = 1.75$

   Putting t = 1.75 into the original equation, gives $d = 30 + 4.2 \times 1.75 - 1.2 \times 1.75^2$

   Maximum height = 33.675 metres

6. $p(x) = 3x^2 - 4x + 8$ $p(6) = 3 \times (6)^2 - 4 \times 6 + 8 = 92$ so point is (6, 92)

   $p'(x) = 6x - 4$ so $p'(6) = 6 \times 6 - 4 = 32$ which is the gradient of the tangent at $x = 6$

   Put into $y - y_1 = m(x - x_1)$ $y - 92 = 32(x - 6)$ **Tangent is $y = 32x - 100$**

Questions 5 and 6 are Merit