L2 Calculus Revision #4

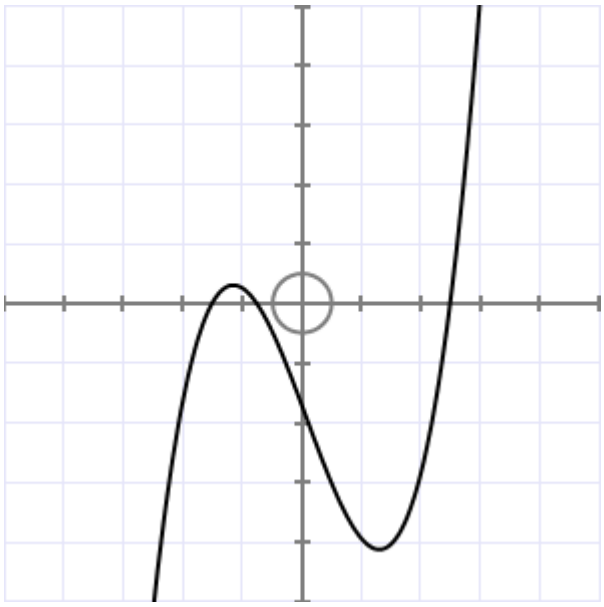
1. Find the gradient of $f(x) = 5x^2 - x^5 + 5$ at $x = 2$.

2. The gradient at any point on a curve is given by $\frac{dy}{dx} = 2x^3 - 4x$

   Find the equation of the curve if it passes through the point (2, 4).

3. To the right is a function $g(x)$.

   Explain clearly where the function would have a negative sign for the gradient function, $g'(x)$.



4. A cubic has equation $y = x^3 - 2x^2 - 2x$.

   Find the points where the slope of the curve is 2.

5. $h(x) = 2x^3 - 18x^2 + 30x + 14$.

   For what values of x is $h(x)$ a decreasing function?

6. The velocity of a ball on elastic goes out from a start point and then back again.

   Its velocity is given by $v(t) = 4(1 - 0.8t)$ where t = time in seconds

   How long is it before the ball returns to its starting point?

Answers: L2 Calculus Revision #4

1. $f(x) = 5x^2 - x^5 + 5$ so $f'(x) = 10x - 5x^4$

   gradient is found from gradient function $f'(2) = 10 \times 2 - 5 \times 2^4 = -60$

   Gradient at $x = 2$ is -60

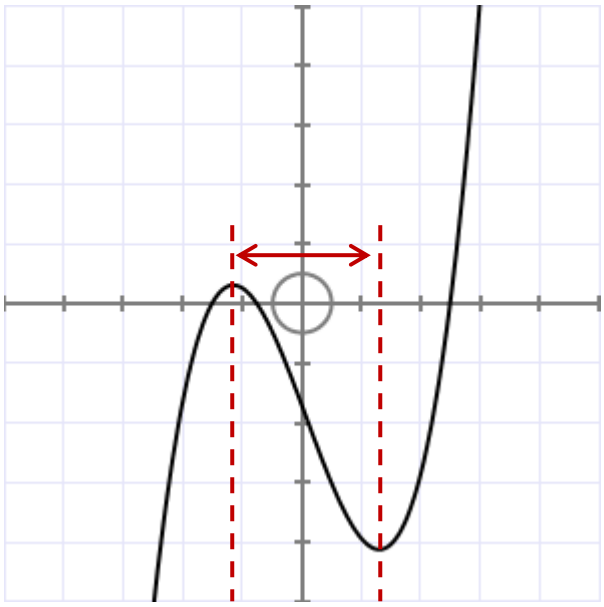
2. $\frac{dy}{dx} = 2x^3 - 4x$ anti-differentiating gives $y = 0.5x^4 - 2x^2 + C$

   Passes through (2, 4) so $4 = 0.5 \times 2^4 - 2 \times 2^2 + C$ So C = 4

   Equation is $y = 0.5x^4 - 2x^2 + 4$

3. The gradient function will be negative **in the range shown**, **not including the turning points** themselves.

   Approximately $-1.1 < x < 1.3$



4. $y = x^3 - 2x^2 - 2x$, so $\frac{dy}{dx} = 3x^2 - 4x - 2$

   We want gradient = 2 so $\frac{dy}{dx} = 2 = 3x^2 - 4x - 2$

   Solving $0 = 3x^2 - 4x - 4$ gives $x = 2$ and -0.6666

   Need the points, so sub into original equation. **Points at (2, –4) and $(\frac{-2}{3}, \frac{4}{27})$**

5. $h(x) = 2x^3 - 18x^2 + 30x + 14$ so $h'(x) = 6x^2 - 36x + 30$

   $h'(x) = 6x^2 - 36x + 30$ has roots at $x = 1$ and 5, so those are turning points

   It is a decreasing function heading down, ⇒ negative slope ⇒ $h'(x) < 0$

   Positive cubic, so down <u>between</u> turning points **Decreasing for $1 < x < 5$**

6. $v(t) = 4(1 - 0.8t) = v(t) = 4 - 3.2t$ so $s(t) = 4t - 1.6t^2 + C$

   Start point can be defined as zero, so C = 0 and we get $s(t) = 4t - 1.6t^2$

   Back at start when s = 0, so $4t - 1.6t^2 = 0$ $t = 0$ and 2.5

   Zero is start time, so return is what we need **Times = 2.5 seconds**

Questions 5 and 6 are Merit