Name:________________________________ Teacher:________________

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St John's College

AS 91256 (v1)

Mathematics and Statistics 2.2

Apply graphical models in solving problems

Data Matching

Credits: 4

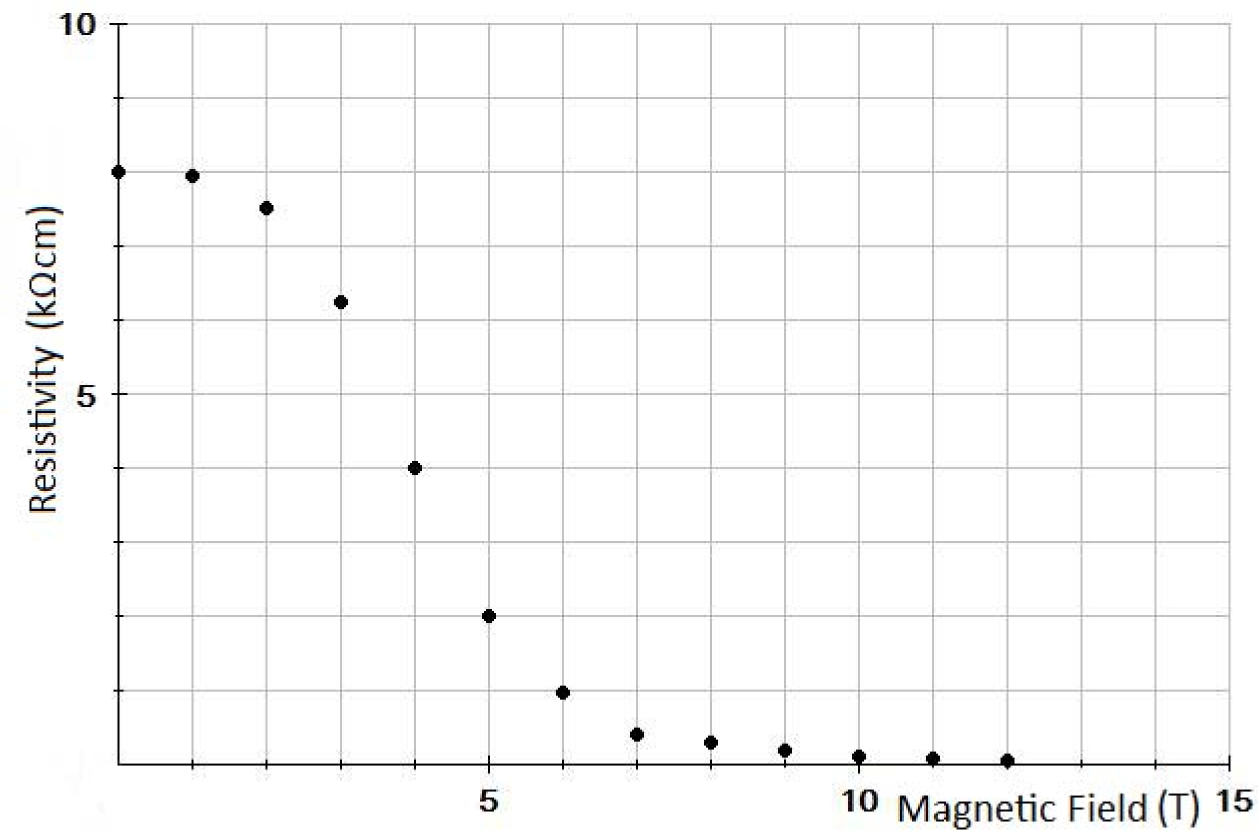
Show ALL working.

YOU MUST HAND YOUR WORK TO THE SUPERVISOR AT THE END OF THE ASSESSMENT.

Achievement	Achievement with Merit	Achievement with Excellence
Apply graphical models in solving problems. ☐	Apply graphical models, using relational thinking, in solving problems. ☐	Apply graphical models, using extended abstract thinking, in solving problems. ☐
		Overall level of performance ☐

Introduction

Peter is researching the resistivity of a ceramic in a magnetic field. He has some data which he has plotted below:



Task

He wants to find the equations to match his data as far as possible, with the following features

a) starting with a slope of zero at $x = 0$,

b) match the data points found as closely as possible, and

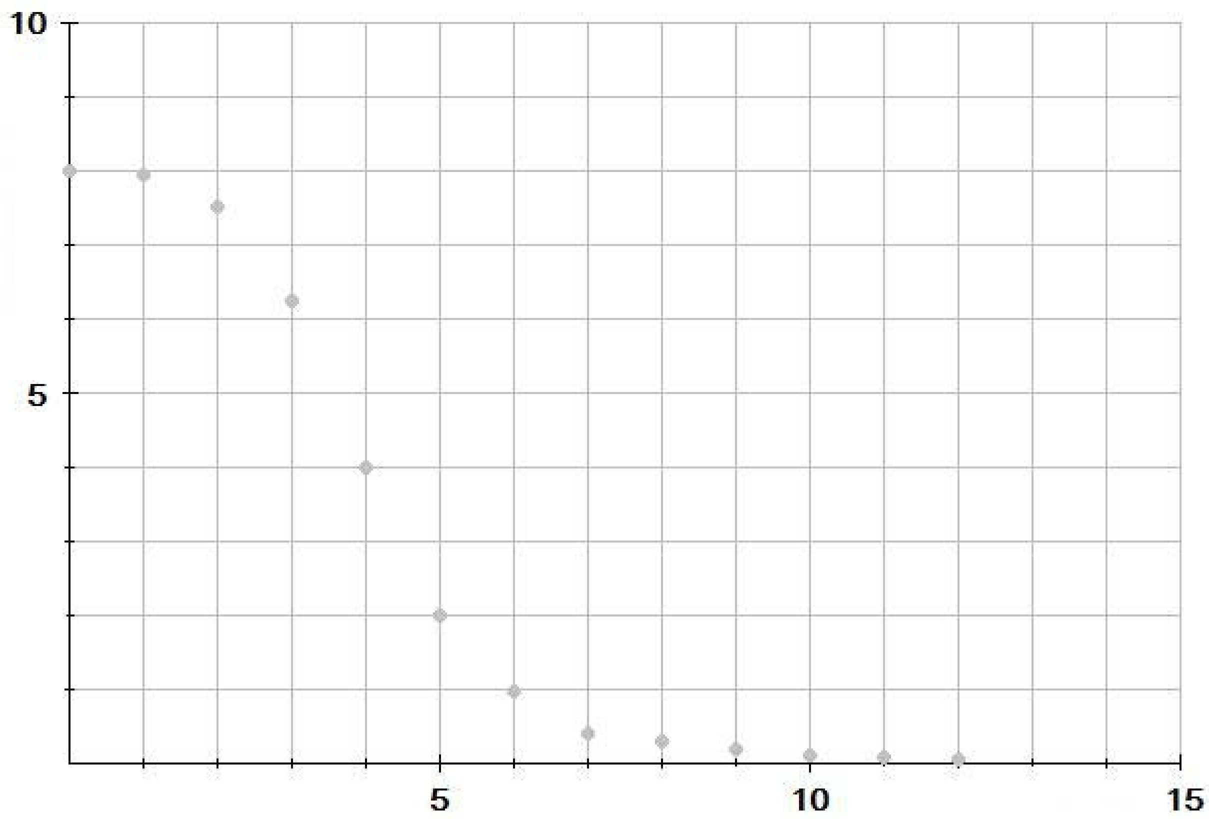
c) never quite reach zero.

For the purposes of curve matching we can ignore the units and label the axes as x and y.

Peter's first attempt at trying to find the equations is:

$$\begin{cases} y = 8 - 0.25x^2 & \text{for } x < 5 \\ y = 1 - \log(x - 5) & \text{for } x \geq 5 \end{cases}$$

Give the features of those graphs. Note how well they meet his three requirements.



a) starting with a slope of zero at $x = 0$ ____________________

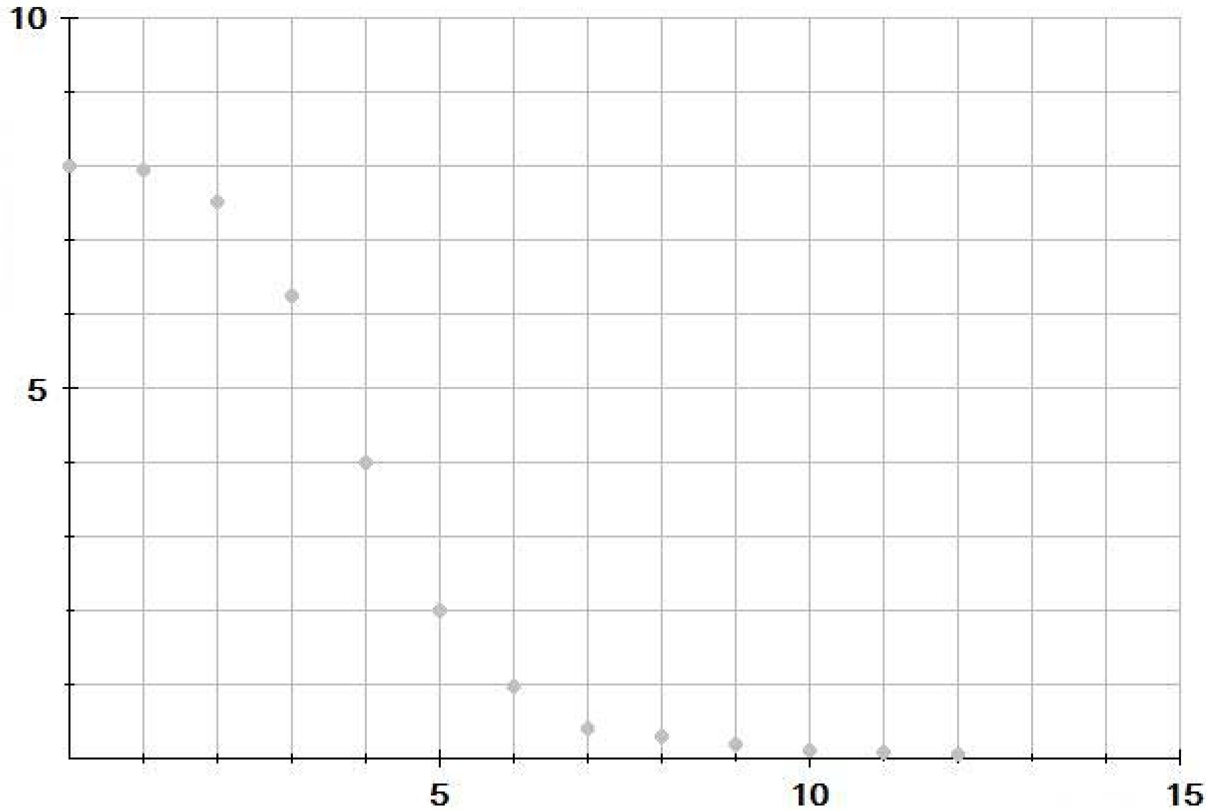
b) match the data points found ____________________

c) never quite reach zero ____________________

Peter's next attempt at trying to find the equations is:

$$\begin{cases} y = 4\cos\left(\frac{\pi x}{8}\right) + 4 & \text{for } x \le 4 \\ y = \frac{4}{x-3} & \text{for } x > 4 \end{cases}$$

State the features of those functions. Note how well they meet his requirements.



a) starting with a slope of zero at $x = 0$ ________________________________

b) match the data points found ________________________________

c) never quite reach zero ________________________________

One option Peter would like to consider is the $y = \sqrt{x}$ function to try and match his experimental x values greater than 4.

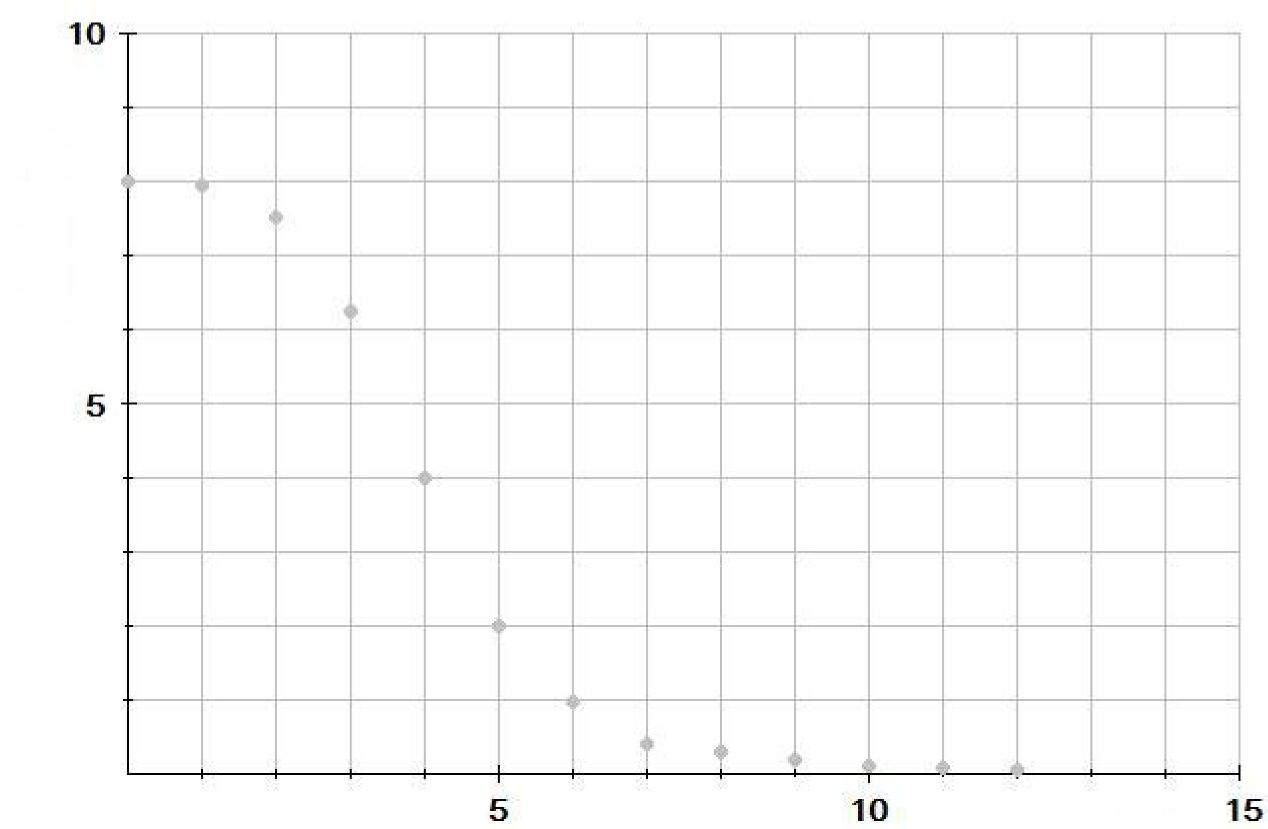
Show how you could attempt to fit that function for $x \geq 5$ given by:

- reflecting it in the x-axis so that it has a negative gradient at all points,
- shifting it an appropriate distance in the y-direction,
- shifting it an appropriate distance in the x-direction, and
- increasing or decreasing the overall gradient for each value.

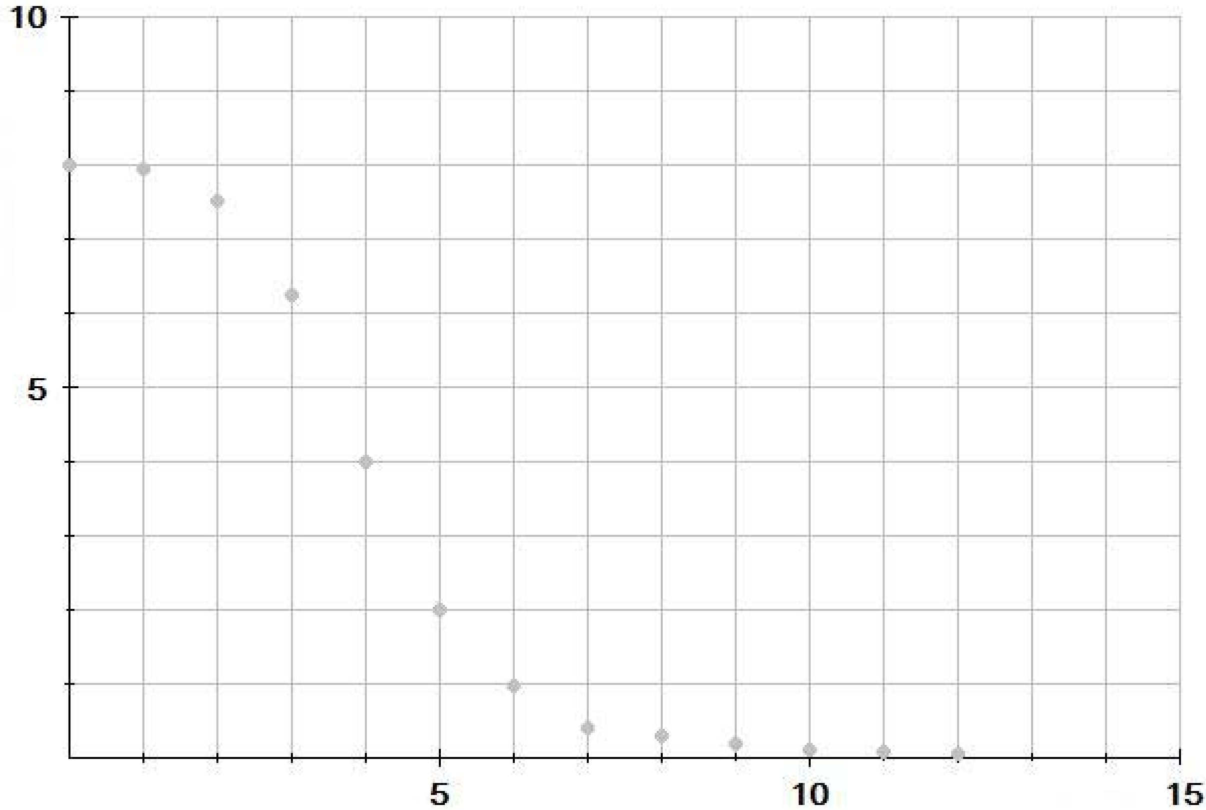
(Note that you do not need to achieve the best possible match, just a reasonable attempt.)

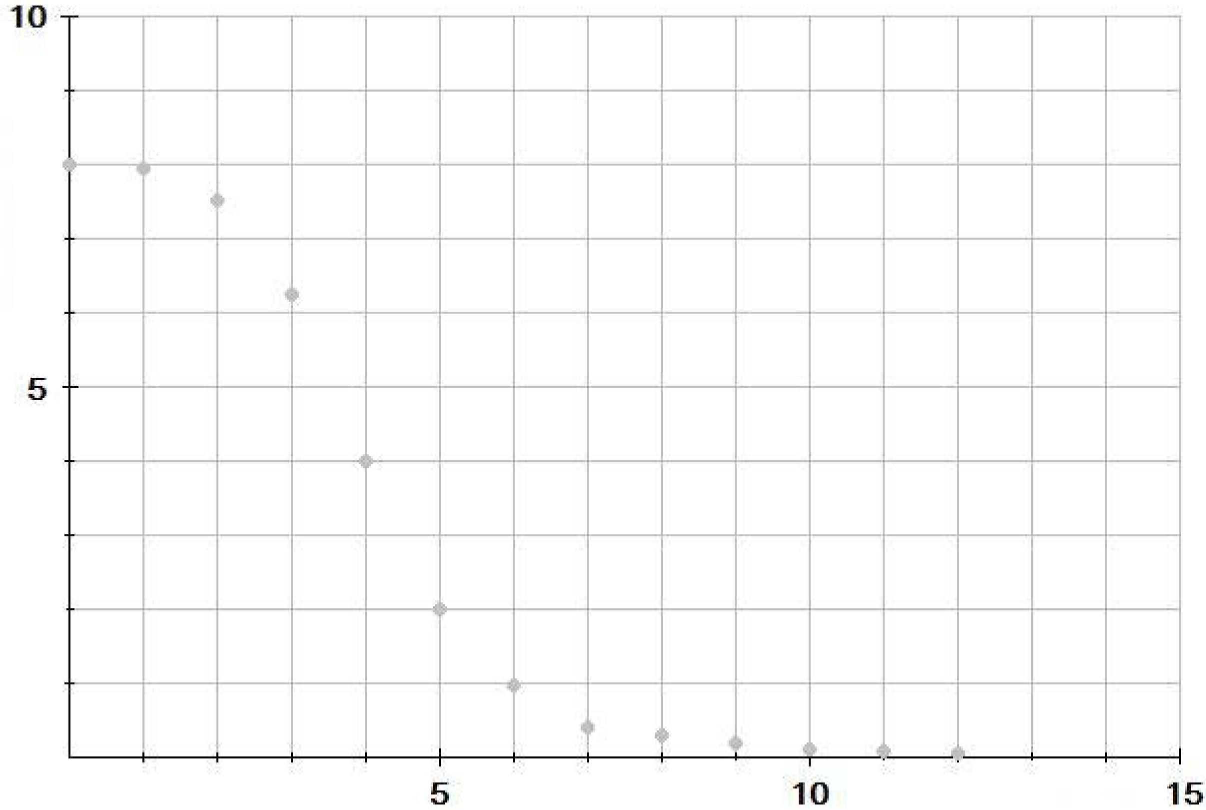
Explore different equations which may match Peter's data better.

(Note credit may be given for incomplete solutions, so show all working)



Spare grids. Please indicate which graphs you are drawing.

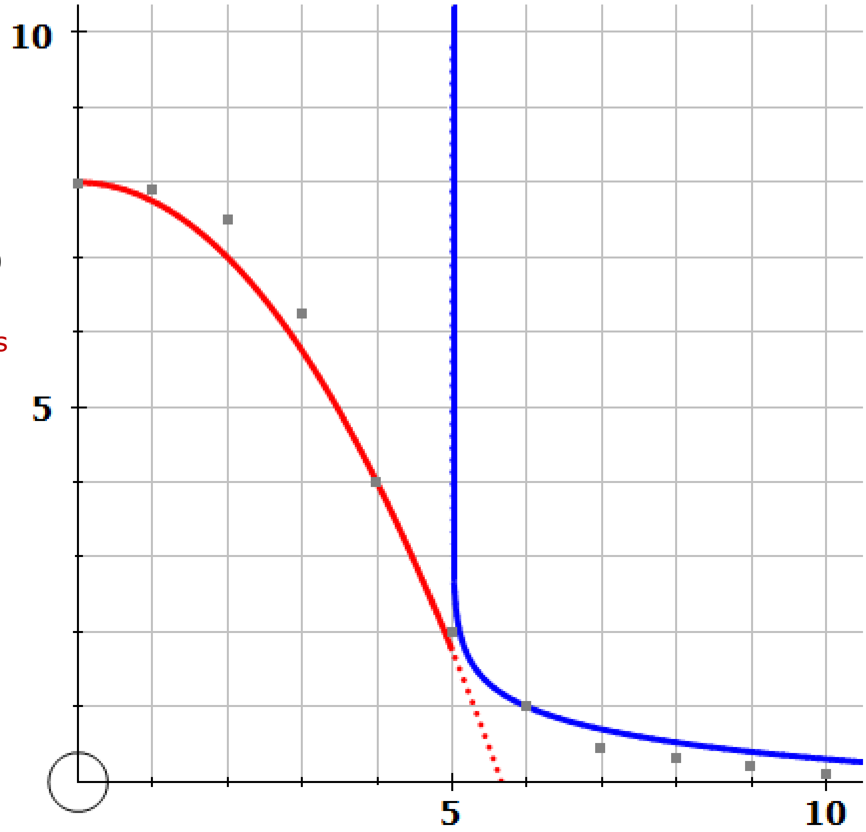




Answers

$y = 8 - 0.25x^2$ for $x < 5$

$y = 1 - \log(x - 5)$ for $x \geq 5$



a) starting with a slope of zero at $x = 0$

parabola has maximum at (0, 8) so this requirement is met.

b) match the data points found

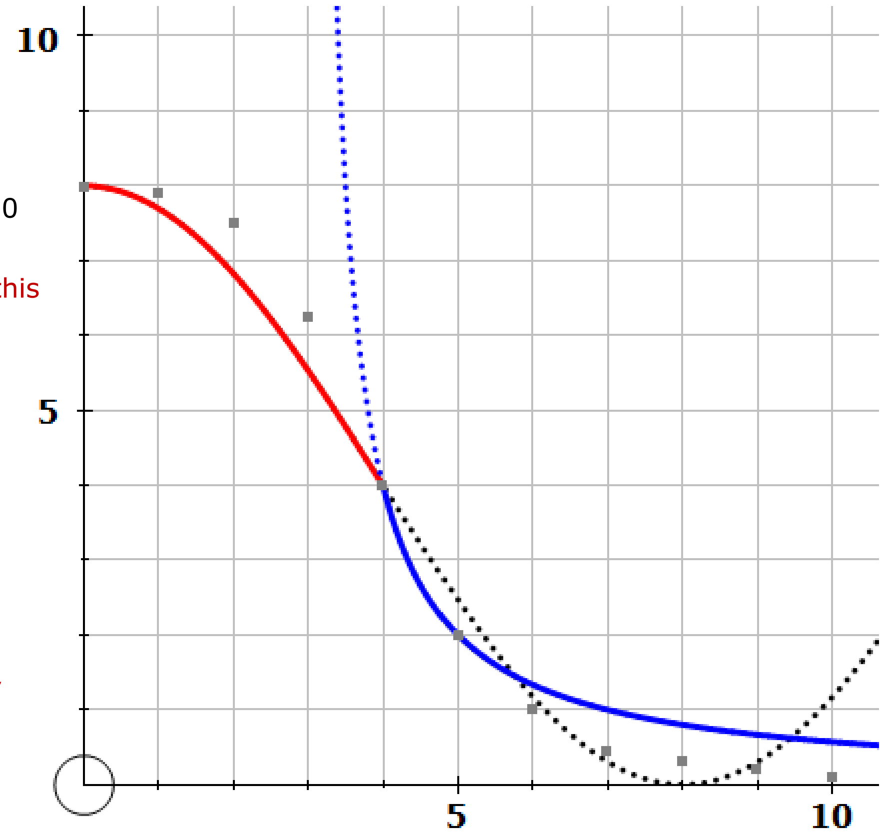
Poor match in general, and totally wrong at x just above 5 where the log graph has an asymptote

c) never quite reach zero

Bad match as the log graph has an intercept at $x = 15$ not an asymptote.

$y = 4\cos\left(\frac{\pi x}{8}\right) + 4$ for $x \leq 4$

$y = \frac{4}{x-3}$ for $x > 4$



a) starting with a slope of zero at $x = 0$

Cos curve has maximum at (0, 8) so this requirement is met.

b) match the data points found

Bad match in general and odd kink in the curve at (4, 4)

c) never quite reach zero

Hyperbola has $y = 0$ as an asymptote, so this requirement is met.

$y = \sqrt{x}$

Show how you could attempt to fit that function for $x \geq 5$ given by:

- reflecting it in the x-axis so that it has a negative gradient at all points

  The function is replaced by its negative

  $y = -\sqrt{x}$ (but definitely not $y = \sqrt{-x}$)

- shifting it an appropriate distance in the y-direction,

  A constant is added to the function to move up in the y direction.

  In this case the end point needs to be moved up two

  $y = -\sqrt{x} + 2$

- shifting it an appropriate distance in the x-direction, and

  Each occurrence of x is replaced by $(x + c)$ to move left in the x direction.

  In this case the end point needs to be moved up five right, so c = −5

  $y = -\sqrt{x-5} + 2$

- increasing or decreasing the overall gradient for each value.

  A multiplier is applied to the (amended) x value to change gradient.

  In this case to flatten it requires a number > 1

  $y = -2\sqrt{x-5} + 2$

For $x \leq 4$

The curve is flatter bottomed than a parabola around (0, 8) which suggests a cubic.

In fact it is $\boldsymbol{y = 8 - \frac{x^3}{16}}$

We could take the parabola that intercepts at $x = 4$, $x = -4$ and $x = -12$ and lift that up 4. That gives $y = -0.2(x - 12)(x - 4)(x + 4) + 4$ which is a relatively good fit, except the maximum is not at $x = 0$. We can fiddle around from there.

y = −0.0144(x−4)(x+5)(x+14) + 4 gives quite a good fit.

For $x \geq 4$

The halving from (4, 4) to (5, 2) and then again to (6, 1) is an indication of a power curve, especially as you need a horizontal asymptote.

The halving means the power must be ½, and then it needs to be shifted to the right.

The actual points are based on $\boldsymbol{y = 0.5^{x-6}}$

A hyperbola fit can be made quite close for x ≥ 5 or 6.

$y = \frac{2}{x-4}$ works relatively well and fiddling makes it closer $\boldsymbol{y = \frac{1.5}{x-4.3}}$ etc

Achieved:

Student applies graphical models in solving problems by demonstrating at least 3 different skills with only minor calculation errors, such as.

a) drawing a graph from an equation

b) identifying key features from an equation near the visible region

c) rewriting an equation to reflect it, or translate in x **or** y direction

In general a student should have at least two correctly drawn graphs including identifying their features (particularly asymptotes) and clear evidence of an ability to translate or transform functions, even if only by way of example.

Merit

Student applies graphical models, using relational thinking by demonstrating a higher level of understanding such as at least 3 different skills from:

d) recognising domain by drawing a correct piecewise graph from an equation

e) identifying key features from an equation well away from the visible region

f) explaining clearly how to translate functions in x **and** y direction

g) recognising how to make a function change in relative gradient

h) writing the equation of a function from points or graph with minor errors

Students clearly explain how graphs are transformed for skills g) and h) – mere examples are insufficient.

Students in general show evidence at Achieved standard in order to reach Merit. A minor omission can be ignored if the Merit level is comfortably shown, but only if skill in three **different** areas is shown. (That is, no matter how good at drawing graphs or finding key features, some other skill is required.)

Excellence

Student applies graphical models, using extended abstract thinking by, such as:

i) recognising the existence of solutions outside the visible region

j) recognise **all** the methods for translating or modifying a function

k) correctly writing the equation of a function from points or graph

Students in general must reach Merit standard in order to reach Excellence, although a minor omission can be ignored if the Excellence level is comfortably shown, if skill in three **different** areas is shown.

Idea taken from *Unusual field dependence of the resistivity and magnetoresistance in* $Nd_{0.5}Ca_{0.5}MnO_3$ which has the following graphs shown: