Name:________________________________ Teacher:________________

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St John's College

AS 91256 (v1)

Mathematics and Statistics 2.2

Apply graphical models in solving problems

Supply and Demand

Credits: 4

Show ALL working.

YOU MUST HAND YOUR WORK TO THE SUPERVISOR AT THE END OF THE ASSESSMENT.

Achievement	Achievement with Merit	Achievement with Excellence
Apply graphical models in solving problems. ☐	Apply graphical models, using relational thinking, in solving problems. ☐	Apply graphical models, using extended abstract thinking, in solving problems. ☐
		Overall level of performance ☐

Introduction

Peter is trying out some supply and demand curves for his economics class.

Task

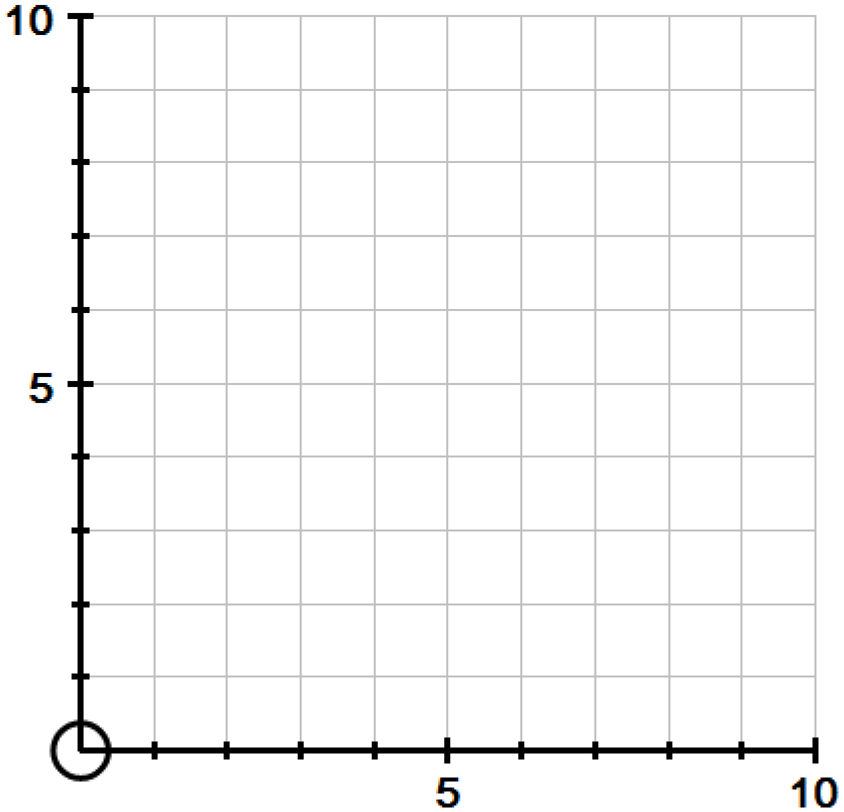
He finds a couple of curves that appear to be suitable.

$y = 2^x - 1$ (supply)

and

$y = \frac{5}{x}$ (demand)

Show the shape of those two functions, and note any features they have (including ones outside the range of the grid shown).



At what point do the two graphs intersect?

Now Peter wants to change the point where the curves intersect.

List **all** the ways that he can move or otherwise alter the supply curve so that the intersection of the two curves is at a greater x value while still keeping the same basic power function shape.

$$y = 2^x - 1 \qquad \text{(supply)}$$

In each case give a numerical example of the altered supply equation and explain what the effect is of the change on the positioning or shape of the curve.

List **all** the ways that he can move or otherwise alter the demand curve so that the intersection of the two curves is at a greater x value while still keeping the same basic power function shape.

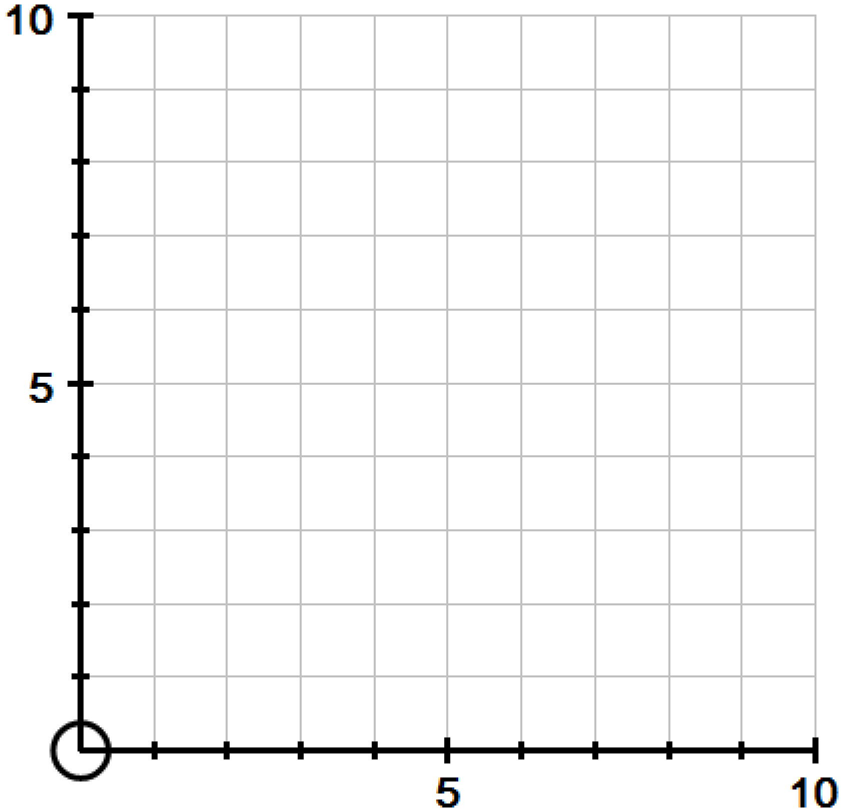
$$y = \frac{5}{x} \qquad \text{(demand)}$$

In each case give a numerical example of the altered supply equation and explain what the effect is of the change on the positioning or shape of the curve.

Another option Peter considered was using piecewise functions.

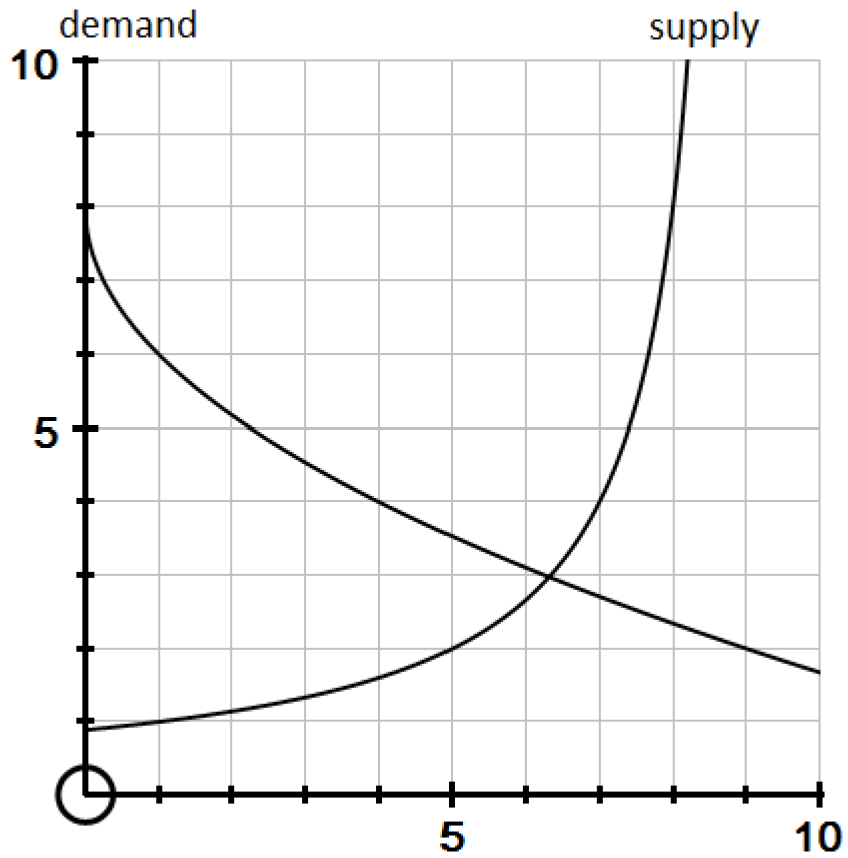
Sketch the following function:

$$\begin{cases} y = (x - 3)^2 + 5 & \text{for } x < 2 \\ y = -5\log(x - 1) + 6 & \text{for } x \geq 2 \end{cases}$$



What features do the individual component functions have that Peter has removed by using a piecewise graph?

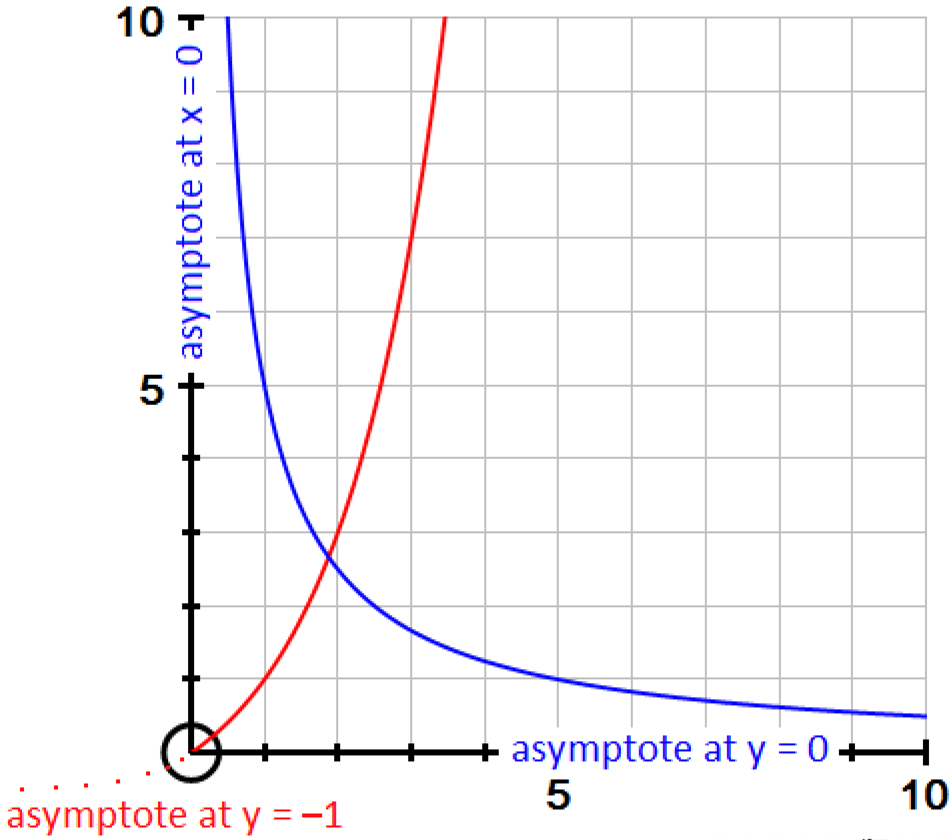
Peter now tries two new functions. The curves he gets are shown below.

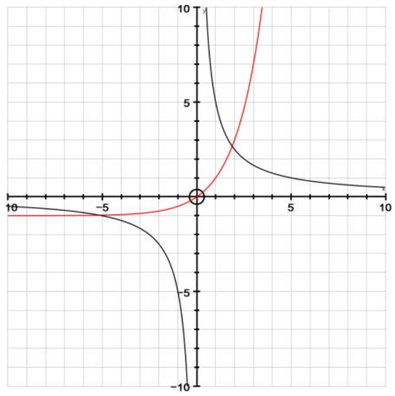


Write the equations that he used.

(Note credit may be given for incomplete solutions, so show all working)

Answers





Graphs intersect at (1.875, 2.667) **and** (−5.145, − 1.028)

Should recognise that there are **two** solutions

$y = 2^x - 1$ can be altered to give a greater x-intersect by:

a) translation in the y-direction by reducing the constant e.g. $y = 2^x - 2$

b) translation in the x-direction by replacing the "x" with "x − shift" e.g. $y = 2^{x-2} - 1$

c) reducing the basic power, e.g. $y = 1.8^x - 1$

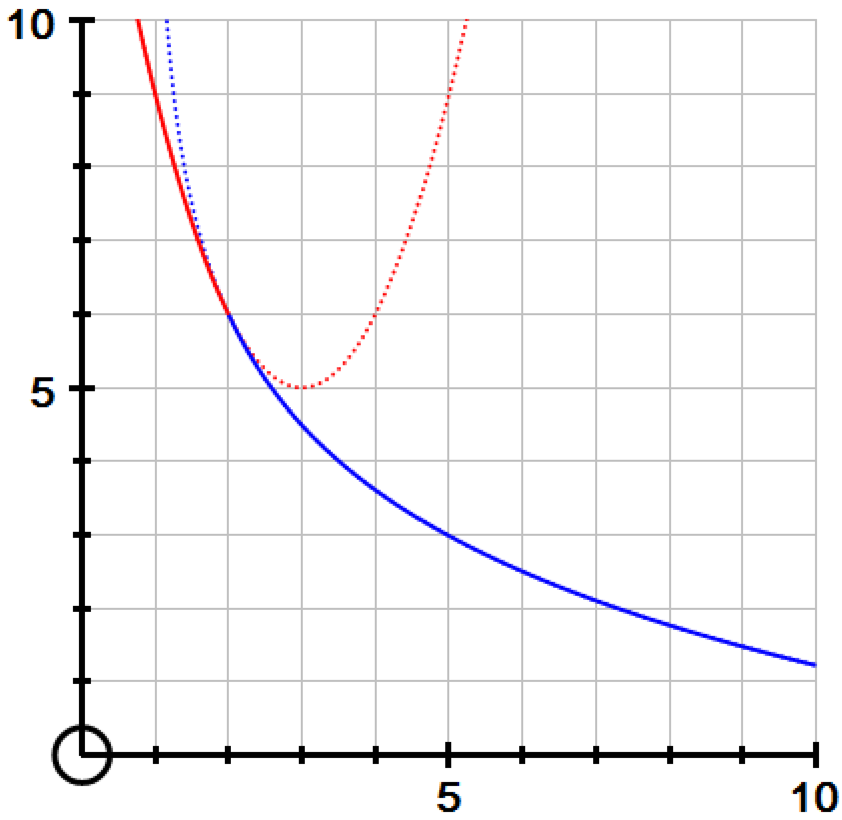
$y = \frac{5}{x}$ can be altered to give a greater x-intersect by:

a) translation in the y-direction by adding a constant e.g. $y = \frac{5}{x} + 2$

b) translation in the x-direction by replacing the "x" with "x − shift" e.g. $y = \frac{5}{x-1}$

c) increasing the numerator, e.g. $y = \frac{8}{x}$

Must not show dotted parts as solid.



log (blue) has

asymptote at x = 1

intercept (16.85, 0)

Parabola (red) has

minimum at (3, 5)

intercept (0, 14)

By making piecewise Peter has removed the turning point and positive curve portion of the parabola and the asymptote of the log graph.

The curves shown are

$\boldsymbol{y = 8 - 2\sqrt{x}}$

which can also be written: $y = {}^{-}2\sqrt{x} + 8$ or $y = 8 - \sqrt{4x}$ or $y = -\sqrt{4x} + 8$ etc

and

$\boldsymbol{y = \frac{-8}{x-9}}$

which can also be written: $y = \frac{8}{9-x}$ etc

Marking Schedule

Achieved:

Student applies graphical models in solving problems by demonstrating at least 3 different skills with only minor calculation errors, such as.

a) drawing a graph from an equation

b) identifying key features from an equation near the visible region

c) rewriting an equation to translate in x **or** y direction

In general a student should have at least two correctly drawn graphs including identifying their features (particularly asymptotes) and clear evidence of an ability to translate or transform functions, even if only by way of example.

Merit

Student applies graphical models, using relational thinking by demonstrating a higher level of understanding such as at least 3 different skills from:

d) recognising domain by drawing a correct piecewise graph from an equation

e) identifying key features from an equation well away from the visible region

f) explaining clearly how to translate functions in x **and** y direction

g) recognising how to make a function change in relative gradient

h) writing the equation of a function from points or graph with minor errors

Students clearly explain how graphs are transformed for skills g) and h) – mere examples are insufficient.

Students in general show evidence at Achieved standard in order to reach Merit. A minor omission can be ignored if the Merit level is comfortably shown, but only if skill in three **different** areas is shown. (That is, no matter how good at drawing graphs or finding key features, some other skill is required.)

Excellence

Student applies graphical models, using extended abstract thinking by, such as:

i) recognising the existence of solutions outside the visible region

j) recognise **all** the methods for translating or modifying a function

k) correctly writing the equation of a function from points or graph

Students in general must reach Merit standard in order to reach Excellence, although a minor omission can be ignored if the Excellence level is comfortably shown, if skill in three **different** areas is shown.