L2 Probability Revision #5

1. Babe Ruth and Ty Cobb were two of the greatest batters in early baseball, but had very different styles. Babe Ruth was famous for his ability to hit hard and long, whereas Ty Cobb was famous for his ability to hit often. Their lifetime statistics are below:

	At Bats	Hits	RBI	Home Runs
Babe Ruth	8,399	2,873	2,231	714
Ty Cobb	11,429	4,191	1,938	117

(*An "at bat" is how many times a player was pitched at, a "hit" means he made it safely to at least first base from an at bat. "RBI" is "runners batted in" which is how many other players already on base scored because of him – and is almost as important as hits.*)

a) Compare the probability for the two batters of getting a "hit" from an "at bat"?

b) Compare the probability for the two players of getting a home run if they scored at least a "hit".

c) Teams will often deliberately "walk" a player who can hit hard. What is the relative risk to the pitcher that Babe Ruth would score a home run compared to Ty Cobb? Interpret that result in terms of which would be better to deliberately walk.

2. A batter has an at bat "hitting average", i.e. probability, of 0.304. Of those hits 22% of are to second base, 12% are to third base, and 14% are home runs.

a) In 90 turns at bat, how many hits to second base or better do you predict he will get?

b) What percentage of hits better than first base are home runs?

c) If he has a hitting average of 0.314 against right-handed pitchers, who make up 80% of pitchers, what is his hitting average against left-handed pitchers?

3. A pitcher has a fast ball that averages 141 kph with a standard deviation of 3.4 kph.

a) What proportion of his fastballs go over 145 kph?

b) How fast are the fastest one-fifth of his fastballs?

c) Assuming the standard deviation remains the same, how much higher does he need to raise his mean so that the fastest one-fifth go over 150 kph?

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Answers: L2 Probability Revision #5

1.

a) P(Cobb hit) = 4191 ÷ 11429 = **0.367** (*which is the highest lifetime average*)
P(Ruth hit) = 2873 ÷ 8399 = **0.342** which seems close, but is 7% difference

b) P(Ruth HR if hit) = 714 ÷ 2873 = **0.248** (*and he holds the "slugging" record still*)
P(Cobb hit) = 117 ÷ 4191 = **0.028** a 10^{th} the rate, so not even remotely close

c) P(Ruth HR if at bat) = $^{714}/_{8399}$ = 0.085. P(Cobb HR if at bat) = $^{117}/_{11429}$ = 0.010
Relative risk for Ruth is 0.085 ÷ 0.010 = **8.3**
As Babe Ruth was 8.3 times as likely to hit a home run, he was a much better prospect to "walk" and avoid the risk of a big hit (*he is 3rd all time for number of walks for pretty much this reason, whereas Cobb was walked only third as much.*)

2.

a) 90 turns at bat = 90 × 0.304 = 27.36 hits.
Of them 22 + 12 + 14 = 48% are to better than first = 27.36 × 0.48 = 13.13.
So predict **13 hits** to 2nd or better.

b) 48% of hits go past 1st, and 14% of hits are homers. P(HR if 2nd) = $^{14}/_{48}$ = **29.2%**

c) The average of 0.304 comes 80% from RH and 20% from LH so we can write
0.8 × 0.314 + 0.2 × x = 0.304 which solves to give x = **0.264** against LH pitchers
(the logic might be better seen with a tree diagram)

3.

a) Graphics normal distribution: Ncd: lower = 145, upper = 99999, σ = 3.4, μ = 141
P(x > 145) = **0.1197**

b) Graphics: InvN: tail = right, area = 0.2, σ = 3.4, μ = 141, = **143 kph**

c) Graphics: InvN: tail = right, area = 0.2, σ = 1, μ = 0, z = **0.8416**
μ + z σ = bound, so μ + 0.8416 × 3.4 = 150, solving gives μ = 147.1
He would need to raise his mean speed by **6.1 kph**.

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