

L2 Probability Practice #2

1. A country has three phone companies with mobile coverage. Their statistics for phone usage (in 1000s) are:

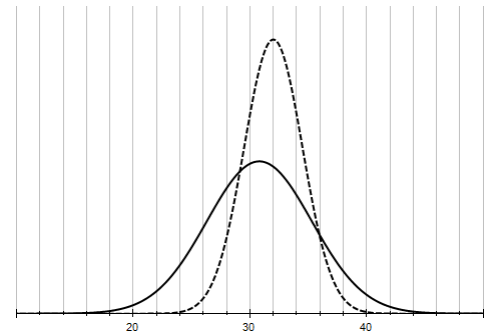
	Telefonica	Voxafone	60 Degrees	Total
Dumb Phones	1711	1665	381	3757
Smart Phones	315	363	72	750
Total	2026	2028	453	4507

- a) What is the probability a randomly selected plan is on 60 Degrees?
- b) What proportion of Telefonica customers have a smart phone?
- c) An analyst says that the rate of smart phone usage with Voxafone customers is over 10% higher than at 60 Degrees. Is this true?
2. The distance at which a particular cell phone tower starts to cut out is measured to have a mean range of 30.8 km with a standard deviation of 4.5 km.
- a) What is the probability a spot 35 km from the tower will be in range?
- b) How close do you have to be to the tower to ensure at least a 95% probability that you are in coverage?
- c) Discuss what the result would be of replacing the tower with another one, that had a mean range of 32 km and a standard deviation of 2.5 km?
3. A cell phone tower is shared by Telefonica and 60 Degrees.
- 85% of the calls are for Telefonica customers.
 - 20% of the calls for Telefonica customers are on smart phones.
 - 12% of the 60 Degrees customers' calls are on smart phones.
- a) What is the probability a random call is a smart phone 60 Degree customers?
- b) If a particular call is selected at random and found to a dumb phone, what is the probability that it is for a 60 Degree customer?
- c) Smart phone calls cost five as much to process as dumb phones for the owners of the tower, due to the extra bandwidth required. What proportion of the tower's costs should Telefonica pay based on usage rates?

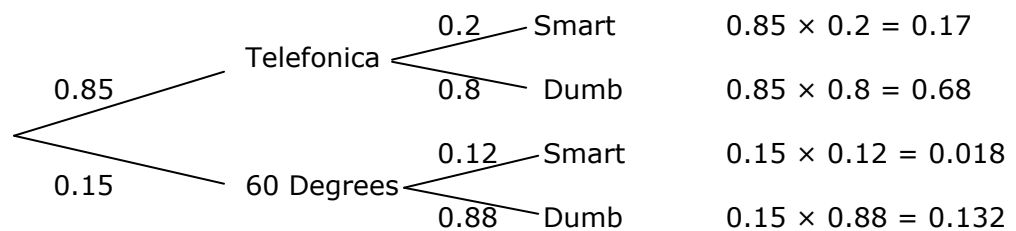
Answers: L2 Probability Practice #2

1. a) 4507 total of which 453 are 60° ⇒ $P(\text{Plan on } 60^\circ) = 453/4507 = \mathbf{0.1005}$
- b) 2026 total at Telefonica, of which 315 have smart phones.
 $P(\text{T customer has smart phone}) = 315/2026 = \mathbf{0.1555}$
- c) Rate at Voxafone = $363/2028 = 0.179$. At 60 Degrees = $72/453 = 0.159$
 Rate of smart phone Voxafone compared to 60° = $0.179/0.159 = \mathbf{1.126}$
True. The Voxafone smart phone rate is **12.6% higher**, which is over 10%.
2. a) Graphics normal distribution: Ncd: lower = 35, upper = 9999, $\sigma = 4.5$, $\mu = 30.8$
 $P(35 > x) = \mathbf{0.175}$ (note: in context Probability **drops** as distance increases)
- b) Graphics normal distribution: InvN: tail = left, area = 0.05, $\sigma = 4.2$, $\mu = 125$
 range = **23.4 km** (95% coverage is same as $P(\text{not covered}) = 0.05$)

- c) The mean distance has increased, so the radius covered is bigger. The standard deviation has decreased, so the variation in coverage is less. This means the distance of almost certain coverage (3σ) is now 24.5 km ($32 - 3 \times 2.5$) which is much better than before at 17.3 km. However there is less chance of coverage at greater ranges. The graph of the situation shows that coverage after 40 km is now almost zero.



3.



- a) $0.15 \times 0.12 = \mathbf{0.018}$
- b) $0.132 + 0.68 = 0.812$ are to dumb phones. Of those 0.132 are to 60°.
 $P(\text{Dumb call to } 60^\circ) = 0.132/0.812 = \mathbf{0.163}$
 (NB: using a 2 way table can make this more obvious, if a bit slower.)
- c) Expected value, with five times weight on the smart phones' usage.
 Telefonica = $5 \times 0.17 + 0.68 = 1.53$ $60^\circ = 5 \times 0.018 + 0.132 = 0.222$
 Telefonica's proportion = $1.53/(1.53 + 0.222) = \mathbf{0.8733} = \mathbf{87.3\% \text{ of costs}}$