Basic Geometric (Multiplication) Sequences

$t_n = a\, r^{\,n-1}$ $\qquad\qquad$ $S_n = \dfrac{a(r^n - 1)}{r - 1}$

1. For the sequence starting with 10 and multiplying by 3 each time:

 a) What value is the 6th term in the sequence?

 b) If we add the first 6 terms, what do they add up to?

2. For the sequence 12, 18, 27, 40.5, … (r = 1.5)

 a) How large is the 20th number in the pattern?

 b) What is the total sum of the first 20 numbers in the pattern?

3. For the sequence 100, 80, 64, … (r = 0.8)

 a) What value is the 15th term in the sequence?

 b) What do all the terms up to the 15th add up to?

4. Peter runs 8 km in the first week. He wants to increase it by 20% each week (r =1.2).

 a) How far would he run in the 15th week if he was to do that?

 b) How far would he have run in total after 12 weeks?

5. A town council spends $600,000 each year on its parks. It wants to decrease its spending by 5% each year (r = 0.95).

 a) How much would the town be spending by the eighth year?

 b) How much would the total spending on parks be after 12 years?

6. Merit: For the sequence 200, 220, 242, 266.2, …

 a) Which term is the first to be more than 400?

 b) If we add them up as we go, when does the total get to 10 000?

Answers: Basic Geometric (Multiplication) Sequences

1. a =10, r = 3, n = 6

   a) $t_6 = 10 \times 3^{\,6-1} = 2430$

   b) $S_6 = \dfrac{10\,(3^6 - 1)}{3 - 1} = 3640$

2. a =12, r = 1.5, n = 20 …

   a) $t_{20} = 12 \times 1.5^{\,20-1} = 26602.05$

   b) $S_{20} = \dfrac{12\,(1.5^{20} - 1)}{1.5 - 1} = 79782.16$

3. a = 100, r = 0.8, n = 15

   a) $t_{15} = 100 \times 0.8^{\,15-1} = 4.398$

   b) $S_{15} = \dfrac{100\,(0.8^{15} - 1)}{0.8 - 1} = 482.4$

4. a = 8, r =1.2, n = 15 and 12

   a) $t_{15} = 8 \times 1.2^{\,15-1} = 102.71$

   b) $S_{12} = \dfrac{8\,(1.2^{12} - 1)}{1.2 - 1} = 316.64$

5. a = \$600,000, r = 0.95, n = 8 and 12

   a) $t_8 = 600000 \times 0.95^{\,8-1} = \$419{,}002$

   b) $S_{12} = \dfrac{600\,000\,(0.95^{12} - 1)}{0.95 - 1} = \$5{,}515{,}679$

6. a = 200, r = 1.1 (220 ÷ 200), n is unknown

   a) $t_n = 400 = 200 \times 1.1^{\,n-1}$ solving, n = 7.27

   so the 8th term will be the first **over** 400

   b) $S_n = 10000 = \dfrac{200\,(1.1^n - 1)}{1.1 - 1}$ solving, n = 18.799

   by the time we get to the 19th term