Sequences and Series Practice #4

$t_n = a + (n - 1)\,d$ $\qquad\qquad$ $t_n = a\,r^{\,n-1}$

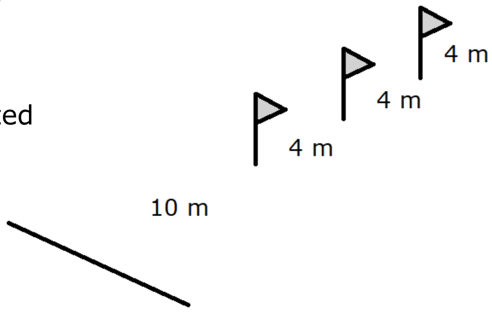
$S_n = \frac{n}{2}\,[2a + (n - 1)d]$ $\qquad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\qquad$ $S_\infty = \frac{a}{1 - r}$

1. Rueben has started powerlifting. He can press 94.5 kg at present. His goal is to increase that by 1.5 kg every month. If he can do that:

   a) How much will he be lifting in 24 months?

   b) How long will it take him until he is lifting 120 kg?

2. A new combine harvester costs $450,000. It is estimated to lose 15% of its value every year. How much will it be worth in ten years' time?



3. Ruby has a sprint training using flags. After she has collected a flag she returns to the start line.

   The first flag is 10 m out. After that they are 4 m apart.

   How many flags will it take her to run at least 800 m?
   (Remember, she runs to the flag and back each time.)

4. A charity collects $20,000 in its first week of a new campaign. The next week it collects only 80% of that ($16,000), and the week after that only 80% of that ($12,800). If the pattern continues:

   a) How much will it raise in the first 10 weeks?

   b) They decide that if they are not getting at least $2,000 in a week that the campaign should be stopped. Advise how long it should run for.

5. One Pacific island has 3,400 people, and is increasing by 2.5% a year. Another has 4,800 people, and is increasing by 1.8% a year. How long till they have the same number of people if those rates remain the same?

Answers: Sequences and Series Practice #4

1. a) How much will he be lifting in 24 months?

$a = 94.5$, $d = +1.5$, want t_{24} $\quad t_n = a + (n - 1)d \;= 94.5 + (24 - 1) \times 1.5$

129 kg

b) How long will it take him until he is lifting 120 kg?

$a = 94.5$, $d = +1.5$, want $t_n = 140$ $\qquad t_n = a + (n - 1)d$

$120 = 94.5 + (n - 1) \times 1.5$

Solving gives **18 months**

2. A new combine harvester costs $450,000. It is estimated to lose 15% of its value every year. How much will it be worth in ten years' time?

$a = 450{,}000$, $r = 0.85$, $n = 10$ $\quad t_{10} = a\,r^{\,n-1} = 450000 \times 0.85^{\,10-1}$

$60,398

3. The first flag is 10 m out. After that they are 4 m apart. How many flags will it take her to run at least 800 m?

$a = 20$, $d = +8$, want $S_n = 800$ $\qquad S_n = \frac{n}{2}[\,2a + (n - 1)d]$

$800 = \frac{n}{2}[\,2 \times 20 + (n - 1) \times 8]$

Solving gives n = 12.28 $\quad$ So on the way to the **13th flag**

4. a) How much will it raise in the first 10 weeks?

$a = 20{,}000$, $r = 0.8$, $n = 10$ $\qquad S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{20000 \times (0.8^{10} - 1)}{0.8 - 1}$

$89263

b) They decide that if they are not getting at least $2,000 in a week that the campaign should be stopped. Advise how long it should run for.

$a = 20000$, $r = 0.8$, want $t_n > 2000$ $\qquad t_n = a\,r^{\,n-1}$

$2000 > 20000 \times 0.80^{\,n-1}$

Solving gives n = 11.31 $\quad$ So should stop after **11 weeks**

5. 3,400 people increasing by 2.5% a year = 4,800 people increasing by 1.8% a year

$t_n = a\,r^{\,n-1} \qquad 3400 \times 1.025^{\,n-1} = 4800 \times 1.018^{\,n-1}$

$3400 \div 4800 = (1.018 \div 1.025)^{\,n-1}$ $\quad$ Solving gives n = 51.3

Soon after 51 years (at a population of around 11774)

Achieved = Q1 a), Q2 and Q4 a). Merit = Q1 b) and Q4 b). Excellence = Q3 and Q5.