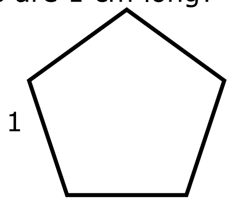
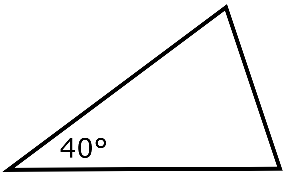
Level 2 Trigonometry Harder #2

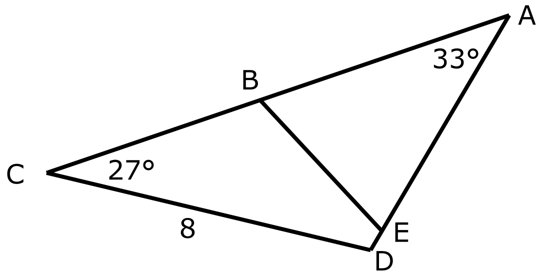
1. What is the area of a regular pentagon where the sides are 1 cm long?



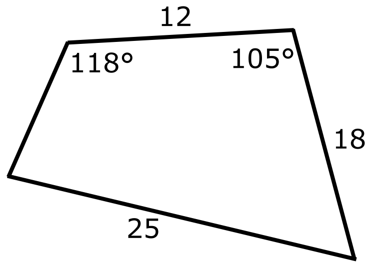
2. An isosceles triangle has an angle of 40° at the non-base angle, and an area of 25 cm^2. What are the side lengths?



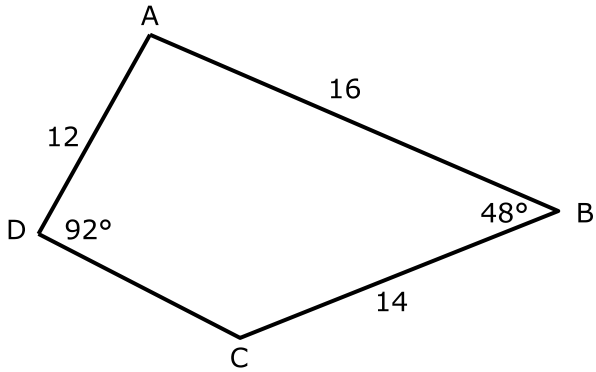
3. Find the distance AB = AE so the triangle ABE is half the area of the triangle ACD



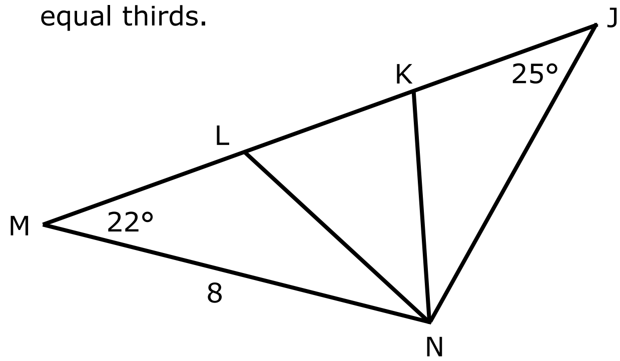
4. what is the area of this quadrilateral?



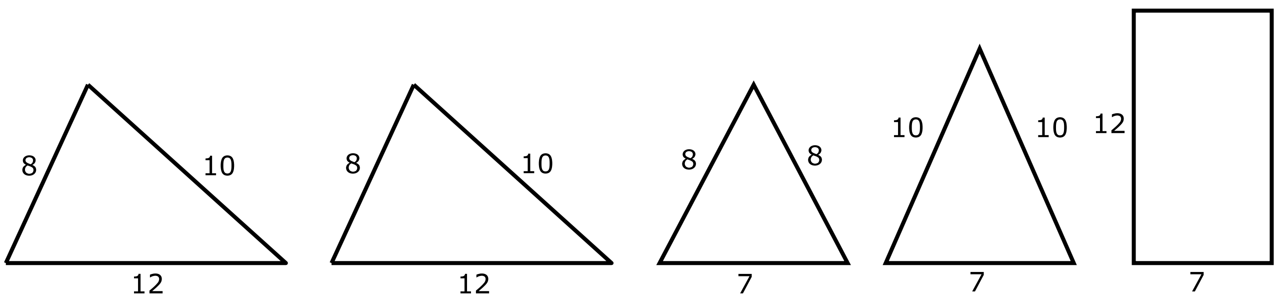
5. What is the distance from D to B?



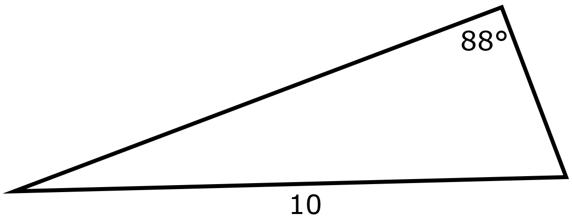
6. Find the angle ∠LKN if the lines KN and LN divide the triangle JMN into equal thirds.



7. An irregular tetrahedron is made from the pieces below. How high is the peak from the base?



8. Find the missing side lengths if the perimeter of the triangle below is 22.

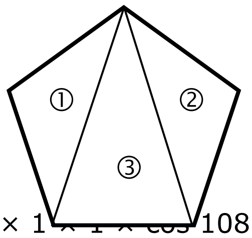


Level 2 Trigonometry Harder #2

Most of the problems can be approached in more than one way, but the methods given here are usually the shortest.

Rounding errors will occur unless you carry all the decimal places.

1. Divide the pentagon into three triangles.



Interior angles are 108° (from 540° ÷ 5)

① Area = ② Area = ½ × 1 × 1 × sin 108° = 0.475

③ The internal triangle sides are 1.618 as $x^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 108° = 2.618$

Angles in the triangle are 36°, 72°, 72° via geometry or Cos Rule

Area = ½ × 1.618 × 1.618 × sin 36° = 0.7694

Total area = 0.475 + 0.475 + 0.770 = 1.72 cm^2

2. From area 25 = ½ × x × x × sin 40°, so the two equal sides are 8.82.

The other side is $d^2 = 8.82^2 + 8.82^2 - 2 \times 8.82 \times 8.82 \times \cos 40°$ d = 6.03 cm

Check: Area = ½ × 8.82 × 6.03 × sin 70° = 25.00, so the answer checks correct.

3. ∠ACD = 180° − 27° − 33° = 120°

AD = 8 ÷ sin 33 × sin 27° = 6.6685 AC = 8 ÷ sin 33 × sin 120° = 12.7207

Area △ACD = ½ × 8 × 12.7207 × sin 27° = 23.100 So area △ABE = 11.550

Let AB = x 11.55 = ½ × x × x × sin 33° ⇒ AB= 6.513

4. Top right triangle (between 12 and 18 sides) Area = ½ × 12 × 18 × sin 105° = 104.32

Now find diagonal across that triangle = x

$x^2 = 12^2 + 18^2 - 2 \times 12 \times 18 \times \cos 105° = 579.81$ $x = \sqrt{579.81} = 24.08$

Angle for left angle of that triangle: $\cos\theta = \dfrac{12^2 + 24.08^2 - 18^2}{2 \times 12 \times 24.08}$ so θ = 46.22°

So the remainder of the 118° for the bottom triangle is 118° − 46.22° = 71.78°

Let y° be the angle diagonally opposite the 105° and z° the other angle in that triangle

$\sin y° = \dfrac{\sin 71.78°}{25} \times 24.08 = 0.9149$ $y° = \sin^{-1}(0.85046) = 66.19°$

$z°$ = 42.07° Bottom triangle, Area = ½ × 25 × 24.08 × sin 42.07° = 201.68

Area of quadrilateral = 201.68 + 104.32 = 306.00

5. $AC^2 = 16^2 + 14^2 - 2 \times 14 \times 16 \times \cos 48°$ $\Rightarrow AC = 12.33813$

$\sin \angle ACB = \sin 48° \div 12.33812 \times 16$ $\Rightarrow \angle ACB = 74.516°$

$\sin \angle ACD = \sin 92° \div 12.33812 \times 12$ $\Rightarrow \angle ACD = 76.410°$

$\sin \angle CAB = \sin 48° \div 12.33812 \times 14$ $\Rightarrow \angle ACB = 74.516°$

$\angle DAC = 180 - 92 - 76.41 = 11.590°$ $\angle DAB = 11.590° + 57.484 = 69.074$

$DB^2 = 16^2 + 12^2 - 2 \times 12 \times 16 \times \cos 69.074$ $\Rightarrow DB = 16.213$

(for reference, if you did it another way, $DC = 2.480$, $\angle DCB = 150.93°$)

6. $\angle MNJ = 180° - 22° - 25° = 133°$

$JN = \dfrac{8}{\sin 25°} \times \sin 22° = 7.091$ $JM = \dfrac{8}{\sin 25°} \times \sin 133° = 13.844$

Area $\triangle JMN = ½ \times 8 \times 7.091 \times \sin 133° -= 20.7445$ (although this isn't required)

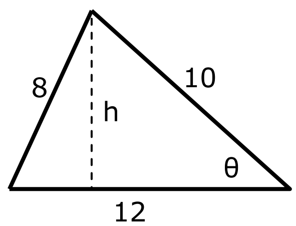
To divide into thirds the line JN is IN equal thirds (as heights same for all) = 4.615

$KN^2 = 4.615^2 + 7.091^2 - 2 \times 4.615 \times 7.091 \times \cos 25° = 12.26$ $KN = \sqrt{12.26} = 3.50$

$LN^2 = 4.615^2 + 8^2 - 2 \times 4.615 \times 8 \times \cos 22° = 16.83$ $LN = \sqrt{16.83} = 4.10$

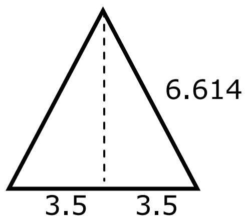
$\angle LKN = \theta$ $\cos\theta = \dfrac{3.5^2 + 4.615^2 - 4{,}1^2}{2 \times 3.5 \times 4.615}$ so $\theta = 58.8°$

7. Taking the scalene triangle, the angle between the 10 and 12 sides is found using



$\cos\theta = \dfrac{10^2 + 12^2 - 8^2}{2 \times 10 \times 12}$ so $\theta = 41.41°$

This means its height $h = \sin 41.41° \times 10 = 6.614$



The cross section of the tetrahedron across the peak is therefore an isosceles triangle of sides 6.614, 6.614 and 7.

It has height (by Pythagoras) of $\sqrt{6.614^2 - 3.5^2}$ = 5.61 high

8. Missing sides can be x and y. $10^2 = x^2 + y^2 - 2 \times x \times y \times \cos 88°$

But perimeter is 22 so $x + y + 10 = 22$ $\Rightarrow y = 12 - x$

$10^2 = x^2 + (12 - x)^2 - 2 \times x \times (12 - x) \times \cos 88$ which is a quadratic

$100 = x^2 + 144 - 24x + x^2 - 0.8376x + 0.0698x^2$ $0 = 2.0698x^2 - 24.8376x + 44$

Solves to give $x = 2.16$ or 9.84 and so $y = 9.82$ or 2.16.