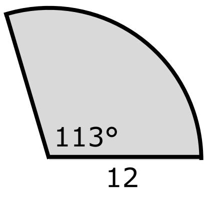
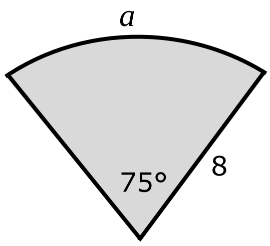
Level 2 Trigonometry Sectors and Segments #2

All curves shown are all parts of circles.

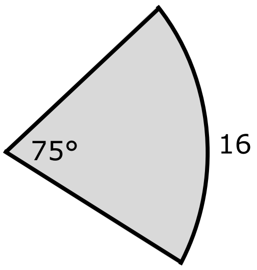
1. Calculate the shaded area shown:



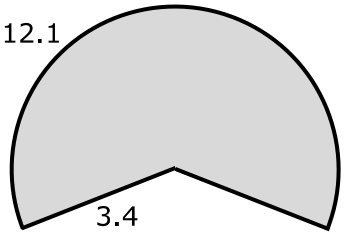
2. Find the the arc length, a.



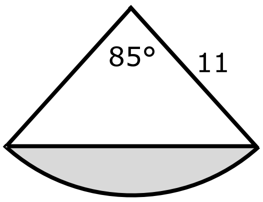
3. The sector's arc length is 16. The angle of the sector is 70°. What is the area of the sector?



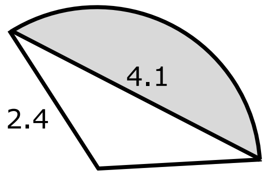
4. The sector's arc is 12.1 long, on a circle of radius 3.4. What is the sector's area?



5. Find the perimeter of the shaded area



6. Calculate the area of the segment.

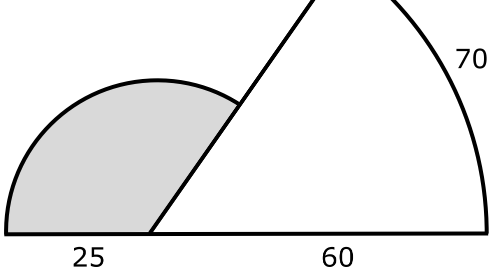


7. Two sectors have the same centre, and their bases form a straight line.

   One is radius 60 m and one is 25 m.

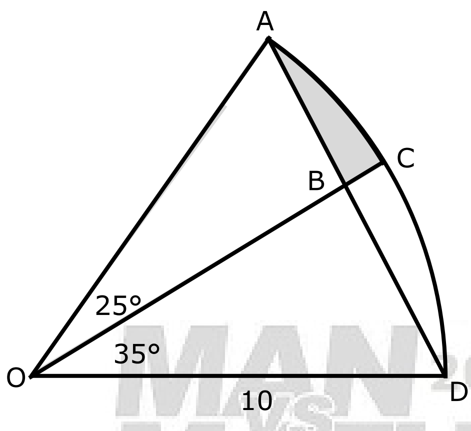
   The arc length of the larger radius one is 70 m.

   What is the shaded area?



8. A segment AD of arc 60° and radius 10 is split in two by a line from the centre to a point 25° along at B.

   What is the perimeter of the shaded area ABC?



Answers: Level 2 Trigonometry Sectors and Segments #2

Rounding errors will occur unless you carry all the decimal places.

1. $A = \frac{113}{360} \times \pi \times 12^2 =$ **142.00**

or

$113° = 113 \times \frac{2\pi}{360} = 1.972$ rad $\quad A = ½\,\theta\, r^2 = 0.5 \times 1.972 \times 12^2 = 142.00$

2. $a = \frac{75}{360} \times \pi \times 2 \times 8 \quad \Rightarrow \quad a =$ **10.47**

or

$75° = 75 \times \frac{2\pi}{360} = 1.309$ rad $\quad a = \theta r = 1.309 \times 8 = 10.47$

3. $75° = \frac{75}{360} = \frac{5}{24}$ of a full circle = 16 long. If 16 is $\frac{5}{24}$ the circumference $= 16 \times \frac{24}{5} = 76.8$

$76.8 = 2\pi r$, so radius $= 76.8 \div 2\pi = 12.22$ $\quad A = \frac{75}{360} \times \pi \times 12.22^2 =$ **96.445**

or

$75° = 75 \times \frac{2\pi}{360} = 1.309$ rad using arc length $= r\theta \quad 16 = r \times 1.309$

$r = 12.22 \quad A = ½\theta r^2 = 0.5 \times 1.309 \times 12.22^2 = 96.445$

4. A circle of radius 3.4 has a circumference of $2 \times 3.4 \times \pi = 21.36$

$\frac{12.1}{21.36} \times \pi \times 3.4^2 =$ **20.57** (the angle is not required, but is $\frac{12.1}{21.36} \times 360 = 203.9°$)

or

using arc length $= r\theta \quad 12.1 = 3.4 \times \theta \quad \Rightarrow \theta = 3.559$ rad

$A = ½\theta r^2 = 0.5 \times 3.559 \times 3.4^2 = 20.57$

5. Arc length $= \frac{85}{360} \times \pi \times 2 \times 11 = 16.32$

line $= \sqrt{(11^2 + 11^2 - 2 \times 11 \times 11 \times \cos(85))} = \sqrt{220.9} = 14.86$

Perimeter = 16.32 + 14.86 = **31.18**

6. To find the angle: $\cos a° = \frac{2.4^2 + 2.4^2 - 4.1^2}{2 \times 2.4 \times 2.4} = \frac{-5.29}{11.52}$ $a° = \cos^{-1}(\frac{-5.29}{11.52}) = 117.34°$

Area sector = $\frac{117.34}{360} \times \pi \times 2.4^2 = 5.898$

Area triangle = ½ × 2.4 × 2.4 × sin(117.34) = 2.558

segment = sector – triangle = 5.898 – 2.558 = **3.34**

7. Make θ the angle at the centre of the white sector. From the arc length

$\frac{\theta}{360} \times \pi \times 2 \times 60 = 70$ $\Rightarrow$ θ = 66.845°

Make Φ the angle at the centre of the grey sector. It is 180 – θ = 113.155°

$\frac{113.155}{360} \times \pi \times 25^2$ = **617.2 m^2**

8. AC = $\frac{25}{360} \times \pi \times 2 \times 10 = 4.363$

Ignoring the centre line, we can solve the angles and side lengths.

AD = $\sqrt{[10^2 + 10^2 - 2 \times 10 \times 10 \times \cos(60)]} = \sqrt{100} = 10$

(or by recognising it as equilateral triangle, or by dividing in half and using RA trig etc)

Therefore ∠OAD = 60° (because equilateral, or by sine rule)

∠OAB = 180 – 60 – 25 = 95°

AB = $\frac{10}{\sin(95)} \times \sin(25) = 4.242$

OB = $\frac{10}{\sin(95)} \times \sin(60) = 8.693$, so BC = 10 – 8.693 = 1.307

perimeter ABC = 4.363 + 4.242 + 1.307 = **9.912**