Differentiation Practice #2

Differentiate

1 $f(x) = 6\tan(3 - 2x)$

2 $y = x^3 . e^{5x}$

3 $y = (x^5 + 2x)^4$

4 $y = \frac{x^2}{\cos x}$

5 $f(x) = 12\,e^{x^2 + 5x}$

6 $y = \sin({}^{-}5t) + \cos({}^{-}5t)$

7 $f(x) = \frac{5x}{\ln(5x)}$

8 $f(x) = 8e^{4 - x} + 17x$

9 $f(x) = \frac{\sqrt{x}}{(x+3)^2}$

10 $f(x) = 4(x^2 + 5x)^3 . \cos(x)$

11 $y = 9 - \sec(5x + 1)$

12 $y = 3\sin^4(4x)$

Answers Differentiation Practice #2

Differentiate | solution | simplified (not required)

1 $f(x) = 6\tan(3 - 2x)$ chain rule of tan(u) so $f'(x) = \sec^2(u).\frac{du}{dx}$ and $\frac{du}{dx} = {}^{-}2$

$f'(x) = 6\sec^2(3 - 2x)\,({}^{-}2)$ $= {}^{-}12 \,.\, \sec^2(3 - 2x)$

2 $y = x^3 .\, e^{5x}$ product f.g so $\frac{dy}{dx}$ = f.g' + f'.g

$\frac{dy}{dx} = x^3 \,.\, 5e^{5x} + 3x^2 \,.\, e^{5x}$ $= x^2 \,.\, e^{5x} \,.\, (5x + 3)$

3 $y = (x^5 + 2x)^4$ chain rule of $(u)^4$ so $\frac{dy}{dx} = 4u^3.\frac{du}{dx}$

$\frac{dy}{dx} = 4(x^5 + 2x)^3 \,.\, (5x^4 + 2)$

4 $y = \frac{x^2}{\cos x}$ quotient rule or product rule of $x^2.\sec(x)$

$\frac{dy}{dx} = \frac{\cos x .2x - x^2 \,.\, (-\sin x)}{\cos^2 x}$ $= \frac{2x\cos x + x^2 \sin x}{\cos^2 x}$

$\frac{dy}{dx} = (x^2) \,.\, (\sec x .\tan x) + (2x)(\sec x)$ $= x\sec x\,(x\tan x + 2)$

5 $f(x) = 12\,e^{x^2 + 5x}$ chain rule of $12\,e^u$ so $f'(x) = 12\,e^u.\frac{du}{dx}$

$f'(x) = 12 \,.\, e^{x^2 + 5x}\,(2x + 5)$ $= 12(2x + 5).e^{x^2 + 5x}$

6 $y = \sin({}^{-}5t) + \cos({}^{-}5t)$ two separate chain rules – note: $\frac{d}{dt}({}^{-}5t) = {}^{-}5$

$\frac{dy}{dt} = \cos({}^{-}5t).({}^{-}5) + -\sin({}^{-}5t)({}^{-}5)$ $= 5\,[\,\sin({}^{-}5t) - \cos({}^{-}5t)\,]$

7 $f(x) = \frac{5x}{\ln(5x)}$ quotient rule where g = ln(5x)= ln(u) so g' $= \frac{1}{u}.\frac{du}{dx}$

$f'(x) = \frac{\ln(5x).(5) - (\frac{1}{5x})(5)(5x)}{(\ln(5x))^2}$ $= \frac{5\ln(5x) - 5}{\ln^2(5x)}$

8 $f(x) = 8e^{4-x} + 17x$ chain rule of $8e^u$ so $f'(x) = 8e^u.\frac{du}{dx}$ and $\frac{du}{dx} = {}^{-}1$

$f'(x) = 8\,e^{4-x}.({}^{-}1) + 17$ $= 17 - 8e^{4-x}$

9 $f(x) = \frac{\sqrt{x}}{(x+3)^2}$ quotient rule: f = $x^{0.5}$ and g = $(x + 3)^2 = u^2$ so g' $= 2u.\frac{du}{dx}$

$f'(x) = \frac{(x+3)^2.(0.5x^{-0.5}) - 2(x+3)(1).(x^{0.5})}{(x+3)^4}$ $= \frac{3 - 3x}{2\sqrt{x}\,(x+3)^3}$

10 $f(x) = 4(x^2 + 5x)^3.\cos(x)$ product f.g so $\frac{dy}{dx}$ = f.g' + f'.g and f' $= 4 \times 3u^2.\frac{du}{dx}$

$f'(x) = (4)[3(x^2 + 5x)^2.(2x + 5)].\cos(x) + (4)(x^2 + 5x)^3(-\sin(x))$

$= (x^2 + 5x)^2.[\,12(2x + 5).\cos x - 4(x^2 + 5x).\sin x\,]$

11 $y = 9 - \sec(5x + 1)$ chain rule of sec(u) so $\frac{dy}{dx} = \sec(u).\tan(u).\frac{du}{dx}$

$\frac{dy}{dt} = 0 - \sec(5x + 1).\tan(5x + 1)(5)$ $= {}^{-}5\sec(5x + 1).\tan(5x + 1)$

12 $y = 3\sin^4(4x)$ chain rule $y = 3u^4$ so $\frac{dy}{dx} = 12u^3.\frac{du}{dx}$ and again u = sin v where v = $4x$

$\frac{dy}{dx} = 3 \,.\, [\,4\sin^3(4x)\,] \,.\, [\,\cos(4x).(4)\,]$ $= 48\sin^3(4x).\cos(4x)$