Differentiation Practice #3

Differentiate

1 $y = (3x^2 + 5x)^3$

2 $y = \sqrt[4]{x^2 + 7x}$

3 $f(x) = 4\,\text{cosec}(5 + 2x)$

4 $f(x) = 12\,e^{x^2 + 5x}$

5 $y = \frac{1}{e^{7x}}$

6 $f(x) = \cos(3x^3 + \ln(x))$

7 $y = \sqrt{x(x+3)}$

8 $f(x) = \frac{5x^2}{x+3}$

9 $y = \ln(5t^2 + 3t)$

10 $f(x) = \tan(x^2 + 5x)$

11 $y = \text{cosec}(4x^3)$

12 $y = (\ln(2x + 3))^2$

Answers Differentiation Practice #3

Differentiate — solution — simplified (not required)

1 $y = (3x^2 + 5x)^3$ — chain rule of $(u)^3$ so $\frac{dy}{dx} = 3u^2 . \frac{du}{dx}$

$\frac{dy}{dx} = 3(3x^2 + 5x)^2 . (6x + 5)$

2 $y = \sqrt[4]{x^2 + 7x}$ — chain rule of $(u)^{0.25}$ so $\frac{dy}{dx} = 0.25u^{-0.75} . \frac{du}{dx}$

$\frac{dy}{dx} = 0.25(x^2 + 7x)^{-0.75} . (2x + 7)$ $= \frac{2x+7}{4\sqrt[4]{(x^2+7x)^3}}$

3 $f(x) = 4 \operatorname{cosec}(5 + 2x)$ — chain rule of cosec(u) so $f'(x) = -\operatorname{cosec}(u).\cot(u).\frac{du}{dx}$

$f'(x) = {}^{-}\operatorname{cosec}(5 + 2x).\cot(5 + 2x).(2)$ $= {}^{-}2 \operatorname{cosec}(5 + 2x) \cot(5 + 2x)$

4 $f(x) = 12 e^{x^2+5x}$ — chain rule of $12 e^u$ so $f'(x) = 12 e^u . \frac{du}{dx}$

$f'(x) = 12 . e^{x^2+5x} (2x + 5)$ $= 12(2x + 5).e^{x^2+5x}$

5 $y = \frac{1}{e^{7x}}$ $= e^{-7x}$ or, much harder, via quotient rule where $\frac{d}{dx}(1) = 0$

$\frac{dy}{dx} = {}^{-}7 e^{-7x}$ $= \frac{-7}{e^{7x}}$

or $\frac{dy}{dx} = \frac{e^{7x} \times 0 - 1 \times 7e^{7x}}{e^{14x}}$ $= \frac{-7}{e^{7x}}$

6 $f(x) = \cos(3x^3 + \ln(x))$ — chain rule of cos(u) so $f'(x) = -\sin(u). \frac{du}{dx}$

$f'(x) = -\sin(3x^3 + \ln(x)).(9x^2 + \frac{1}{x})$

7 $y = \sqrt{x(x+3)}$ — chain rule: $u^{0.5}$ where $u = x^2 + 3x$ so $\frac{dy}{dx} = 0.5u^{-0.5} . \frac{du}{dx}$

$\frac{dy}{dx} = 0.5 (x^2 + 3x)^{-0.5}.(2x + 3)$ $= \frac{2x+3}{2\sqrt{x^2+3x}}$

8 $f(x) = \frac{5x^2}{x+3}$ — quotient rule or product rule of $(5x^2)(x + 3)^{-1}$

$f'(x) = \frac{(x+3)(10x)-(1)(5x^2)}{(x+3)^2}$ $= \frac{5x(x+6)}{(x+3)^2}$

or $f'(x) = (5x^2)({}^{-}1(x + 3)^{-2}.(1)) + (10x)(x + 3)^{-1}$ $= {}^{-}5x^2(x + 3)^{-2} + 10x(x + 3)^{-1}$

9 $y = \ln(5t^2 + 3t)$ — chain rule: ln(u) so $\frac{dy}{dx} = \frac{1}{u}. \frac{du}{dx}$

$\frac{dy}{dt} = \frac{1}{5t^2+3t} (10t + 3)$ $= \frac{10t+3}{5t^2+3t}$

10 $f(x) = \tan(x^2 + 5x)$ — chain rule: tan(u) so $\frac{dy}{dx} = \sec^2 u. \frac{du}{dx}$

$f'(x) = \sec^2(x^2 + 5x).(2x + 5)$ $= (2x + 5) \sec^2(x^2 + 5x)$

11 $y = \operatorname{cosec}(4x^3)$ — chain rule of cosec(u) so $\frac{dy}{dx} = -\operatorname{cosec}(u).\cot(u). \frac{du}{dx}$

$\frac{dy}{dx} = -\operatorname{cosec}(4x^3).\cot(4x^3).(12x^2)$ $= {}^{-}12x^2 \operatorname{cosec}(4x^3).\cot(4x^3)$

12 $y = (\ln(2x + 3))^2$ — chain rule $y = u^2$ so $\frac{dy}{dx} = 2u.\frac{du}{dx}$ and $u = \ln(v)$ where $v = 2x+3$

$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx} = [2 \ln(2x + 3)].[\frac{1}{2x+3}].[2]$ $= \frac{4\ln(2x+3)}{2x+3}$