Calculus Polar Complex Number Practice #1

1. Write $k + 5ki$ in polar form.

2. $w = n \text{ cis } (\frac{\pi}{8})$ and $v = 2n \text{ cis } (\frac{5\pi}{8})$. Calculate the exact value of wv.

3. $w = n \text{ cis } (\frac{\pi}{8})$ and $v = 2n \text{ cis } (\frac{5\pi}{8})$. Calculate the exact value of $\frac{w}{v}$.

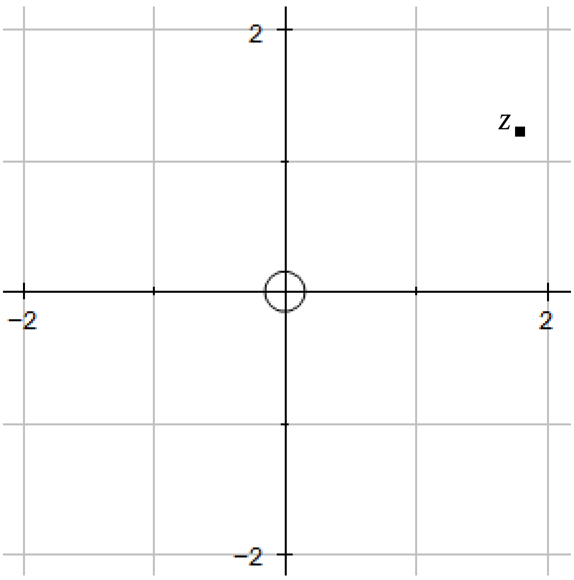
4. What are k and x if $z = k \text{ cis } (3.5) = x - 5i$?

5. Find z such that $z^3 = 3 - 3i$.

6. $z = k \text{ cis } (\frac{n\pi}{8})$. For what integer values of n is z^5 a real number?

7. Write $^{-}\sqrt{48} + 4i$ in exact polar form.

8. Plot the square roots of z on the diagram:



Answers: Calculus Rectangular Complex Number Practice #1

1. Write $k + 5ki$ in polar form.

$|z| = \sqrt{(k)^2 + (5k)^2}$ arg $z = \tan^{-1}(\frac{5k}{k}) = \tan^{-1}(5)$ $\boldsymbol{z = \sqrt{26}\, k \text{ cis } (1.3734)}$

or, for when $k < 0$ $z = \sqrt{26}\, k \text{ cis } (1.3734 + \pi) = \boldsymbol{\sqrt{26}\, k \text{ cis } (4.515)}$

2. $w = n \text{ cis } (\frac{\pi}{8})$ and $v = 2n \text{ cis } (\frac{5\pi}{8})$. Calculate the exact value of wv.

$w.v = (n \times 2n) \text{ cis } (\frac{5\pi}{8} + \frac{\pi}{8})$ $= \boldsymbol{2n^2 \text{ cis } (\frac{3\pi}{4})}$ $(= {}^{-}\sqrt{2}\, n^2 + \sqrt{2}\, n^2\, i)$

3. $w = n \text{ cis } (\frac{\pi}{8})$ and $v = 2n \text{ cis } (\frac{5\pi}{8})$. Calculate the exact value of $\frac{w}{v}$.

$\frac{w}{v} = (\frac{n}{2n}) \text{ cis } (\frac{5\pi}{8} - \frac{\pi}{8})$ $= \boldsymbol{0.5 \text{ cis } (\frac{\pi}{2})}$ $(= 0.5\, i\,)$

4. What are k and x if $z = k \text{ cis } (3.5) = x - 5i$?

$k \cos(3.5) + k \sin(3.5)\, i = x - 5i$ so $k = {}^{-}5 \div \sin(3.5)$ $\Rightarrow \boldsymbol{k = 14.25}$

and $x = k \cos(3.5) = 14.25 \cos(3.5)$ $\Rightarrow \boldsymbol{x = {}^{-}13.35}$

5. Find z such that $z^3 = 3 - 3i$.

$z^3 = 3 - 3i = \sqrt{18} \text{ cis } (\frac{7\pi}{4})$ or $z^3 = 3 - 3i = \sqrt{18} \text{ cis } (\frac{-\pi}{4})$

$z = \sqrt[3]{\sqrt{18}} \text{ cis } (\frac{7\pi}{4} \div 3)$ by De Moivre's Theorem, with solutions at $\frac{2\pi}{3} n$

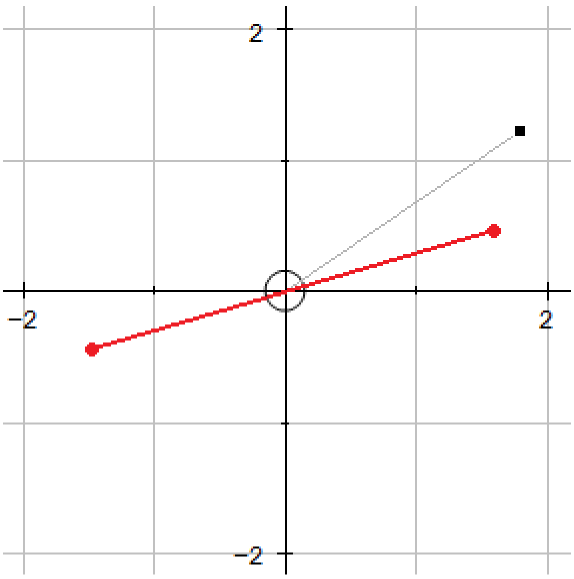
$\boldsymbol{z_1 = \sqrt[6]{18} \text{ cis } (\frac{7\pi}{12})}$ $\boldsymbol{z_2 = \sqrt[6]{18} \text{ cis } (\frac{15\pi}{12})}$ $\boldsymbol{z_3 = \sqrt[6]{18} \text{ cis } (\frac{23\pi}{12})}$ $= \sqrt[6]{18} \text{ cis } (\frac{-\pi}{12})$

6. $z = k \text{ cis } (\frac{n\pi}{8})$. For what integer values of n is z^5 a real number?

$z^5 = k^5 \text{ cis } (\frac{5n\pi}{8})$ by de Moivre, so z^5 is real when $\frac{5n\pi}{8} = x\pi$ where x is an integer

So $n = \frac{8}{5}x$ where $x \in \mathbb{Z}$ but only want integer n, so need to get rid of fraction

$\boldsymbol{n = 5x}$ **where** $\boldsymbol{x \in \mathbb{Z}}$ or in words, n is any integer multiple of 5 (includes negatives)



7. Write ${}^{-}\sqrt{48} + 4i$ in exact polar form.

${}^{-}\sqrt{48} + 4i = 4({}^{-}\sqrt{3} + 1i)$ which is from

$\frac{\pi}{6}$ / $\frac{\pi}{3}$ / $\frac{\pi}{2}$ triangle $= \boldsymbol{8 \text{ cis } (\frac{5\pi}{6})}$

8. Plot the square roots of z on the diagram:

Arg(z) halved, modulus about $\sqrt{2}$

Second solution at $\frac{2\pi}{2} = \pi$ away