Calculus Polar Complex Number Practice #2

1. Write the polar form for a negative imaginary number, ^{-}ki .

2. Solve $z^4 + 5 = 0$.

3. $v = k$ cis $(\frac{3\pi}{5})$. Calculate the exact value of v^{-1}.

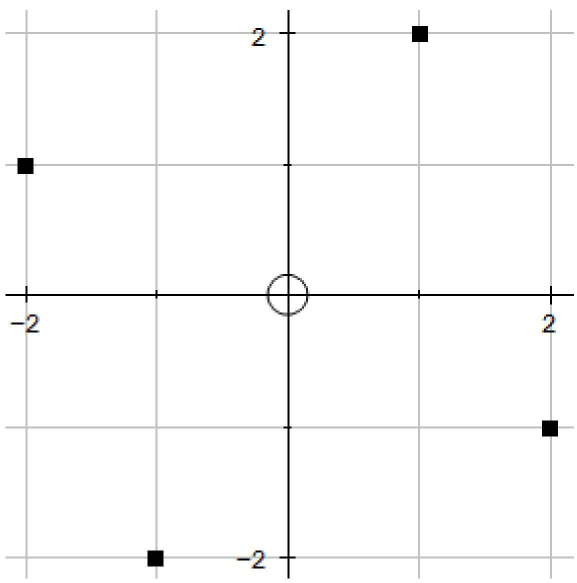
4. If $w = n$ cis $(\frac{\pi}{6})$ find v so that $\frac{w}{v} = 5i$.

5. What are x and θ if $z = 5$ cis (θ) $= x + 4i$?

6. Calculate $(k + \sqrt{-k^2})^8$ in simplest form

7. $z = 7.2$ cis $(\frac{\pi}{6})$. For what integer values of n is z^n a purely imaginary number?

8. Write an equation for which the four points shown are the solutions.



Answers: Calculus Rectangular Complex Number Practice #2

1. Write the polar form for a negative imaginary number, ^{-}ki .

   modulus k, negative imaginary is $\frac{-\pi}{2}$ axis (down) **k cis ($\frac{-\pi}{2}$) or k cis ($\frac{3\pi}{2}$)**

2. Solve $z^4 + 5 = 0$.

   $\Rightarrow z^4 = {}^{-}5$ $z^4 = 5 \text{ cis } (\pi)$ $z = \sqrt[4]{5} \text{ cis } (\pi \div 4)$ by De Moivre

   $z_1 = \sqrt[4]{5}$ cis ($\frac{\pi}{4}$) **$z_2 = \sqrt[4]{5}$ cis ($\frac{3\pi}{4}$)** **$z_3 = \sqrt[4]{5}$ cis ($\frac{5\pi}{4}$)** **$z_4 = \sqrt[4]{5}$ cis ($\frac{7\pi}{4}$)**

3. $v = k \text{ cis } (\frac{3\pi}{5})$. Calculate the exact value of v^{-1}.

   $v^{-1} = \frac{1}{v} = \frac{1 \text{ cis } (0)}{k \text{ cis } (\frac{3\pi}{5})} = (\frac{1}{k}) \text{ cis } (0 - \frac{3\pi}{5})$ $\Rightarrow v^{-1} =$ **k^{-1} cis ($\frac{-3\pi}{5}$)** or $\frac{1}{k}$ cis ($\frac{7\pi}{5}$)

4. If $w = n \text{ cis } (\frac{\pi}{6})$ find v so that $\frac{w}{v} = 5i$.

   $\frac{w}{v} = 5$ means $v = \frac{w}{5i} = \frac{n \text{ cis } (\frac{\pi}{6})}{5 \text{ cis } (\frac{\pi}{2})} = (\frac{n}{5}) \text{ cis } (\frac{\pi}{6} - \frac{\pi}{2}) =$ **$\frac{n}{5}$ cis ($\frac{-\pi}{3}$)** or **$\frac{2\pi}{3}$**

5. What are x and θ if $z = 5 \text{ cis } (\theta) = x + 4i$?

   $|z| = 5 = \sqrt{x^2 + 4^2}$ $\Rightarrow$ **$x = 3$ or $^{-}3$**

   $\pm 3 = 5 \cos(\theta)$ $\Rightarrow \theta = \cos^{-1}(\frac{3}{5})$ and $\pi - \cos^{-1}(\frac{3}{5})$ $\Rightarrow$ **θ = 0.9273 or 2.2143**

6. Calculate $(k + \sqrt{-k^2})^8$ in simplest form

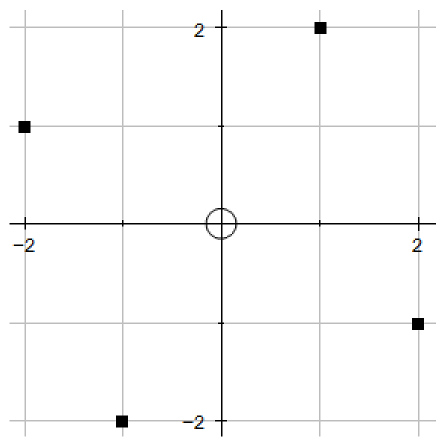
   $k + \sqrt{-k^2} = k + ki = \sqrt{2}k \text{ cis } (\frac{\pi}{4})$ $\Rightarrow (k + \sqrt{-k^2})^8 = (\sqrt{2}k)^8 \text{ cis } (\frac{8\pi}{4}) =$ **$16k^8$**

7. $z = 7.2 \text{ cis } (\frac{\pi}{6})$. For what integer values of n is z^n a purely imaginary number?

   $z^n = 7.2^n \text{ cis } (\frac{n\pi}{6})$ by de Moivre, so z^n is imaginary when $\frac{n\pi}{6} = \frac{\pi}{2}$ or $\frac{-\pi}{2}$

   So $n = 3$ or $^{-}3$. But also solutions at 2π from those $\Rightarrow$ **$n = 3 + 6x$ where $x \in \mathbb{Z}$**

8. Write an equation for which the four points shown are the solutions.



Four points, so it is of the form $z^4 = k$.

One answer is $1 + 2i$, so $k = (1 + 2i)^4$

$z^4 = (1 + 2i)^4$ or **$z^4 = {}^{-}7 + 24i$**

or alternatives like $z^4 + 7 - 24i = 0$