Calculus Rectangular Complex Number Practice #1

1. Give the roots of $x^2 + 2x + 21$ in exact values, using surds if required.

2. Give the roots of $x^2 - 2kx + 10k^2$, using surds if required.

3. What is the remainder when $5x^3 - 40x^2 + 14x + 18$ is divided by $x - 5$?

4. Show, using the Factor Theorem, that $x - 3$ is a factor of $x^3 - 3x^2 + x - 3$.

5. For what values of k does $3x^2 + kx + 10$ have only complex roots?

6. $6i$ is a root of $2x^3 + 3x^2 + kx + 108$. Find the other two roots.

7. Write $\frac{2k + 3i}{ki}$ in the form $a + bi$

8. Calculate z if $\frac{5\bar{z}}{z} = 4 - 3i$ and $|z| = \sqrt{40}$

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Answers: Calculus Rectangular Complex Number Practice #1

1. Give the roots of $x^2 + 2x + 21$ in exact values, using surds if required.

$\frac{-2 \pm \sqrt{2^2-4\times1\times21}}{2\times1}$ $= \frac{-2}{2} \pm \frac{\sqrt{-80}}{\sqrt{4}}$ $= ^-1 \pm \sqrt{20}\,i$ $= ^-1 \pm 2\sqrt{5}\,i$

2. Give the roots of $x^2 - 2kx + 10k^2$, using surds if required.

$\frac{--2k \pm \sqrt{(2k)^2-4\times1\times10k^2}}{2\times1} = \frac{2k}{2} \pm \frac{\sqrt{-36k^2}}{\sqrt{4}}$ $= k \pm 3k\,i$

3. What is the remainder if $5x^3 - 40x^2 + 14x + 18$ is divided by $x - 5$?

$f(5) = 5(5)^3 - 40(5)^2 + 14(5) + 18 = ^-287$ so the remainder is $^-287$

4. Show, using the Factor Theorem, that $x - 3$ is a factor of $x^3 - 3x^2 + x - 3$.

$f(3) = (3)^3 - 3(3)^2 + (3) - 3 = 0$ so $x - 3$ is a factor

5. For what values of k does $3x^2 + kx + 10$ have only complex roots?

complex roots if $\Delta = b^2 - 4ac < 0$

$k^2 - 4 \times 3 \times 10 < 0$, so $k^2 < 120$ $^-\sqrt{120} < k < \sqrt{120}$

6. $6i$ is a root of $2x^3 + 3x^2 + kx + 108$. Find the other two roots.

Factor theorem $2(^-6i)^3 + 3(^-6i)^2 + k(^-6i) + 108 = 0$ NB: $f(6i) = 0$ as well

Imaginary parts need to equal zero: $2(^-6i)^3 + k(^-6i) = 0$ $\Rightarrow k = 72$

$2x^3 + 3x^2 + 72x + 108$ has roots (graphics calculator) of $6i$, ^-6i and $^-1.5$

7. Write $\frac{2k + 3i}{ki}$ in the form $a + bi$

$\frac{(2k + 3i)i}{ki \times i}$ $= \frac{2ki + 3i^2}{-k}$ $= \frac{-3 + 2ki}{-k}$ $= \frac{3}{k} - 2i$

8. Calculate z if $\frac{5\bar{z}}{z} = 4 - 3i$ and $|z| = \sqrt{40}$

Let $z = a + bi$. So $\frac{5\bar{z}}{z} = \frac{5(a - bi)}{a + bi} = 4 - 3i$ $\Rightarrow 5a - 5bi = (4 - 3i)(a + bi)$

$5a - 5bi = 4a + 3b - 3ai - 3bi$ matching real parts: $5a = 4a + 3b$ $a = 3b$

$|z| = \sqrt{40}$ $\Rightarrow a^2 + b^2 = 40$ $\Rightarrow (3b)^2 + b^2 = 40$ $\Rightarrow b = \pm2$

$\Rightarrow z = 6 + 2i$ or $z = ^-6 + ^-2i$

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