

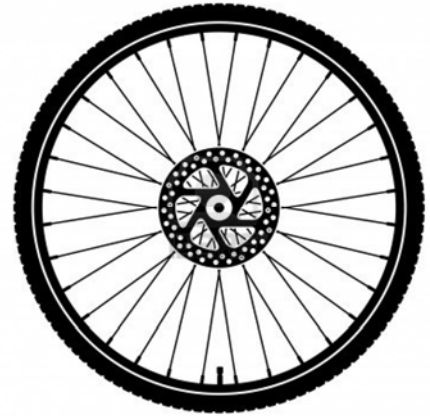
Calculus Trigonometry Practice #2

1. A wheel rotates at 4 revolutions per minute.

The diameter of the wheel is 85 cm.

At $t = 0$ a point on the wheel is 10 cm above the ground and is rising.

Write an expression for the height of that point from the ground over time.



2. A population is thought to follow a sine curve.

It reaches a maximum of 50,000 on Day 12.

It falls to the next minimum of 38,000 on Day 76.

- What is the population on Day 200?
- On which day after Day 400 is the population next at a minimum?
- How long on each cycle is the population above 40,000?

3. A sinusoidal pattern is such that

a value of 32 is reached at $t = 10$,

a value of 8 at $t = 26$,

a value of 32 again at $t = 42$, and

a value of 32 again at $t = 58$

Write a trigonometric equation that fits these criteria.

Answers: Calculus Trigonometry Practice #2

Solutions may be done with different trig equations from those shown.

1. Amplitude = 42.5 (the radius). The midpoint = 42.5 (centre of wheel)

Period = 15 seconds. $H = 42.5 \sin\left(\frac{2\pi}{15}(t - t_0)\right) + 42.5$ is the basic form

At $t = 0$, height = 10, and solving that gives $t_0 = 2.08$, which is on the way up

$$H = 42.5 \sin\left(\frac{2\pi}{15}(t - 2.08)\right) + 42.5 \quad \text{or} \quad H = 42.5 \cos\left(\frac{2\pi}{15}(t - 5.83)\right) + 42.5$$

2. Amplitude = $(50000 - 38000) \div 2 = 6000$. The midpoint = $(50000 + 38000) \div 2 = 44000$

Period = $2 \times (76 - 12) = 128$ days First peak is at Day 12.

$$P = 6000 \cos\left(\frac{2\pi}{128}(t - 12)\right) + 44000 \quad \text{or} \quad P = 6000 \sin\left(\frac{2\pi}{128}(t + 20)\right) + 44000$$

a) $P_{200} = 6000 \cos\left(\frac{2\pi}{128}(200 - 12)\right) + 44000 = \mathbf{38,115}$

- b) Cycles of 128 days from Day 76 gives minima at Days 76, 204, 332 and **Day 460**.

c) Solving $40000 = 6000 \cos\left(\frac{2\pi}{128}(t - 12)\right) + 44000$ gives 58.87

Rounding gives Day 59. This is a down-slope solution. Next down slope is 187.

Up-slope is the other side of the trough at $t = 76$, which gives Day 93.

From Day 93 to day 187 = **94 days in each cycle**.

3. Day 26 is mid way between the equal values of 10 and 42, so this must be the minimum.

The period is 48 days (= 58 - 10). From minima we can write negative cos equation:

$$y = -A \cos\left(\frac{2\pi}{48}(t - 26)\right) + C \quad \text{where } C - A = 8$$

Using our known value: $32 = -A \cos\left(\frac{2\pi}{48}(10 - 26)\right) + 8 + A$ from which $A = 16$

$$y = -16 \cos\left(\frac{2\pi}{48}(t - 26)\right) + 24 \quad (\text{or equivalents})$$