Calculus Trigonometry Practice #4

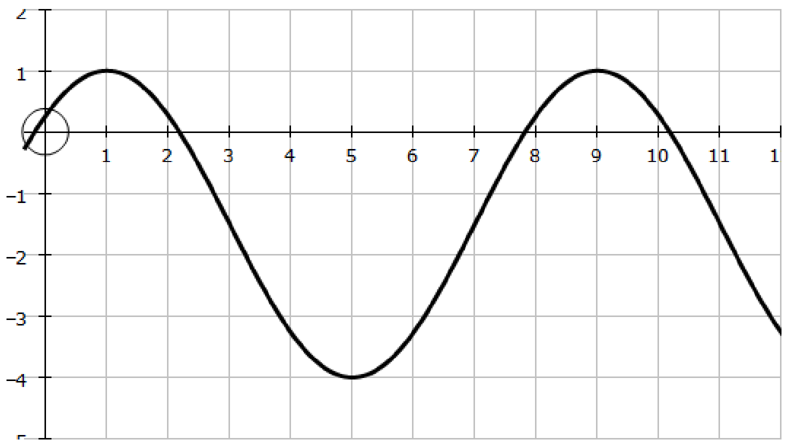
1. A harbour has high tides that are 12 hours and 24 minutes apart.

   A bridge over the harbour is such that the maximum clearance (at low tide) is 5.8 metres and the minimum clearance (at high tide) is 2.6 metres.

   Construct a model for the clearance under the bridge relative to some start time $t = 0$ measured in hours.

   Use that model to show how long in each tide that the clearance is at least 3 metres..

2. Write four different equations for the graph shown to the right (i.e. not just shifted one period each time).



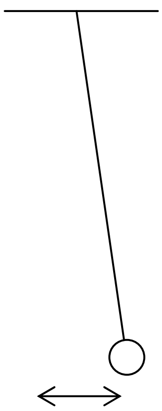
3. A pendulum swings back and forth with minimal friction.

   The pendulum moves to a maximum of 3 cm from directly vertical on both sides.

   It is at its left-most point at 1.1 seconds.

   It is next at its right-most point at 3.8 seconds.

   Construct a general solution for the times the pendulum is at least 1 cm away from directly vertical.



Answers: Calculus Trigonometry Practice #4

Solutions may be done with different trig equations from those shown.

1. Amplitude = (5.8 – 2.6) ÷ 2 = 1.6 Midpoint = (5.8 + 2.6) ÷ 2 = 4.2

   Period = 12.4 hours. The start point is not given so set first peak at t = 0.

   C = 1.6 cos $\left(\frac{2\pi}{12.4}t\right)$ **+ 4.2** (NB: sine graph will be exactly the same.)

   Solving for C = 3 gives solutions of 4.7737 and 7.6263.

   But these solutions span the trough, not the crest, so moving the first one forward 12.4, gives 17.1737 and 7.6263 hours (which is symmetrical across the crest at 12.4 hours).

   Each cycle is above 3 metres for 17.1737 – 7.6263 = **9.55 hours**.

2. Amplitude = (1 – $^{-}4$) ÷ 2 = 2.5 Midpoint = (1 + $^{-}4$) ÷ 2 = $^{-}1.5$

   Period = 9 – 1 = 8 (from peak to peak). Shift = 1 to peak for cos graph

   y **= 2.5 cos** $\left(\frac{2\pi}{8}(t-1)\right)$ **– 1.5** y **= 2.5 sin** $\left(\frac{2\pi}{8}(t-7)\right)$ **– 1.5**

   y **=** $^{-}$**1.5 – 2.5 cos** $\left(\frac{2\pi}{8}(t-5)\right)$ y **=** $^{-}$**1.5 – 2.5 sin** $\left(\frac{2\pi}{8}(t-3)\right)$

3. Amplitude = 3 Midpoint = 0

   Period= (3.8 – 1.1) × 2 = 5.4 seconds. Offset = 1.1 for peak (if we count left as +)

   Y **= 3 cos** $\left(\frac{2\pi}{5.4}(t-1.1)\right)$ **if + left** or Y = $^{-}3$ cos $\left(\frac{2\pi}{5.4}(t-3.8)\right)$ if right is +

   Solving for Y = 1 gives $t = \cos^{-1}\left(\frac{1}{3}\right) \times \frac{5.4}{2\pi} + 1.1 =$ **2.158** sec, down slope, and 0.042 on the opposite side of the maximum at 1.1. These solutions repeat every 5.4 seconds.

   >1 to left when **0.042 + 5.4*n* < t < 2.158 + 5.4*n*** where $n \in \mathbb{Z}$ (i.e. n is an integer)

   Solving for Y = $^{-}1$, which is solution to the right gives $t = \cos^{-1}\left(\frac{-1}{3}\right) \times \frac{5.4}{2\pi} + 1.1 =$ **2.742** sec, up slope, and 4.858 on the opposite side of the minimum at 3.8.

   >1 to right when **2.742 + 5.4*n* < t < 4.858 + 5.4*n*** where $n \in \mathbb{Z}$