Calculus Trigonometry Practice #5

1. An old fashioned water wheel rotates by the movement of water pushing against its blades.

   The axle of the water wheel (i.e. its centre of rotation) is 1.2 metres above the water, the radius of the waterwheel is 1.5 metres and it turns every 40 seconds.

   Model the height of a point on the end of a blade relative to the water level.

   How long does that point spend in the water for each rotation?

2. A gardener records the temperature in a heated hot house and finds it follows a sine curve.

   The temperature averages 28°, reaching a maximum of 32° at 3:20 p.m.

   When is the temperature above 29.5º?

3. Below is a table of average monthly temperatures at Wellington Airport from 1971-2000

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
°C	17.8	17.9	16.6	14.4	12.0	10.2	9.5	9.9	11.3	12.9	14.5	16.4

   Graph this data.

   Write a trigonometric equation using the cosine function that you feel best models how to find the average temperature for any day of the year.

   Find which days of the year you would predict the average temperature would exceed 14°.

   Explain how using a sine function might give different answers. Which function is better?

   (taken from www.niwa.co.nz/education-and-training/schools/resources/climate/modelling)

Answers: Calculus Trigonometry Practice #5

Solutions may be done with different trig equations from those shown.

1. Amplitude = 1.5 (the radius). The midpoint = 1.2 (height of axle)

   Period = 40 seconds. Make water level H = 0, so vertical offset is 1.2

   $\boldsymbol{H = 1.5 \sin\left(\frac{2\pi}{40}t\right) + 1.2}$ **or** $\boldsymbol{H = 1.5 \cos\left(\frac{2\pi}{40}t\right) + 1.2}$

   Under water when H < 0 $t = \sin^{-1}\left(\frac{-1.2}{1.5}\right) \times \frac{40}{2\pi} = {}^{-}\mathbf{5.9}$ on up-slope

   Need down-slope <u>before</u> that. Other side of trough at $^{-}10$ seconds = $^{-}$**14.1 seconds**

   Difference in times is **8.2 seconds** each rotation

2. $\boldsymbol{T = 4 \cos\left(\frac{2\pi}{24\times60}(t - 920)\right) + 28}$ T is temperature, t is time in minutes

   Solve: $29.5 = 4\cos\left(\frac{2\pi}{1440}(t - 920)\right) + 28$

   $t = \cos^{-1}\left(\frac{1.5}{4}\right) \times \frac{1440}{2\pi} + 920 = \mathbf{1191.9}$ This is down-slope solution.

   Up-slope is the other side of the peak, $t = 920 - (920 - 1191.9) = \mathbf{648.1}$

   Converting to normal times those are **10:48 a.m. to 7:52 p.m.**

3. Temperatures are plotted in the **middle** of each month, as they span across that range.

   $T = 4.4 \cos(2\pi/365\,(d - 24)) + 13.7$ fits for the start of the year – shown first next page

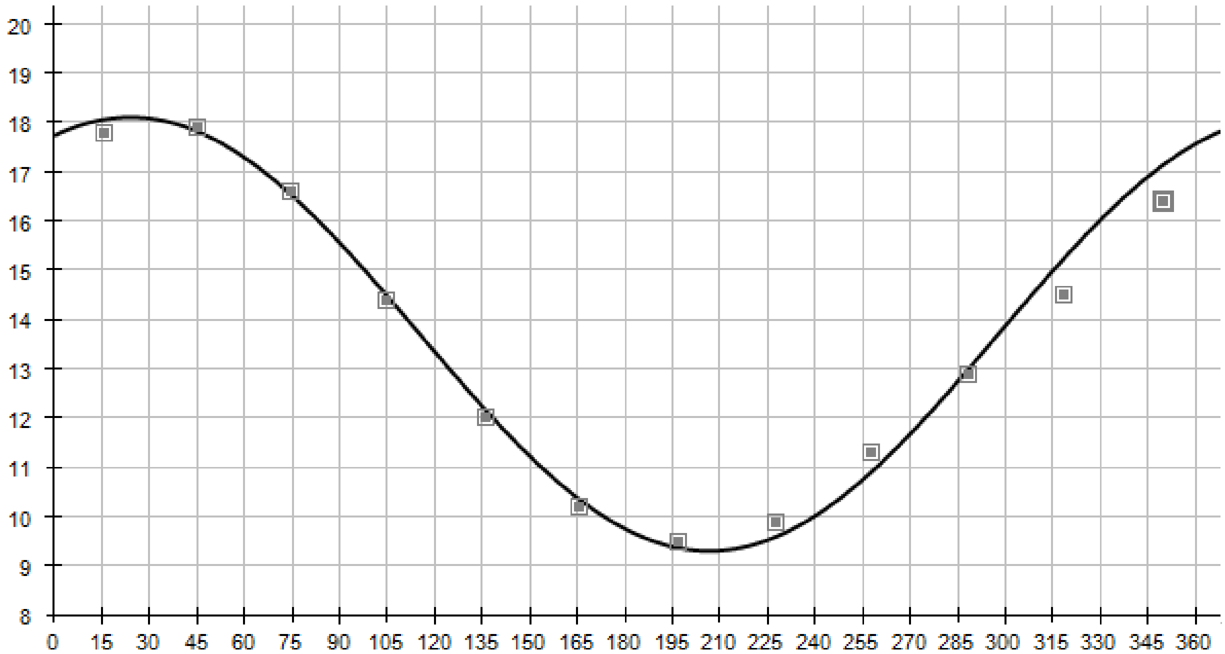
   $T = 4.0 \cos(2\pi/365\,(d - 23)) + 13.5$ fits for the end of the year – shown second next page

   Combining the two: $T = 4.2 \cos(2\pi/365\,(d - 24)) + 13.6$ is as close as it gets overall.

   Note that the amplitude can exceed the variation from 9.5° to 17.9° because the average for the month will hide the highest and lowest days.

   The “C” shift need not be exactly to a month either, just whatever works to give the best fit.

   Using a Sine function gives identical results.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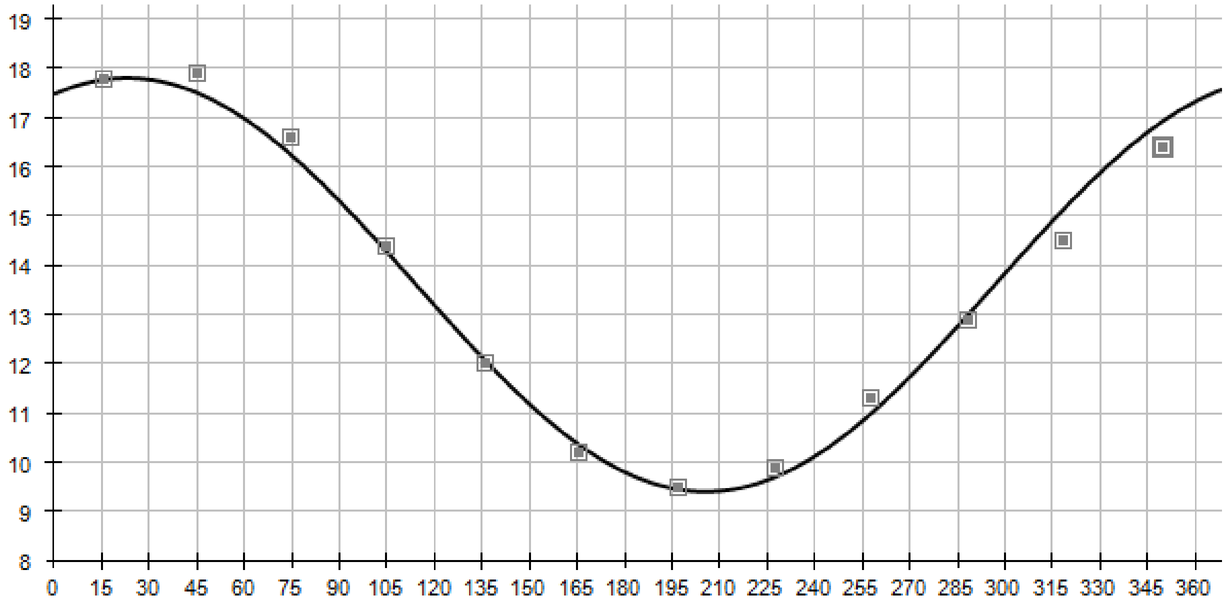
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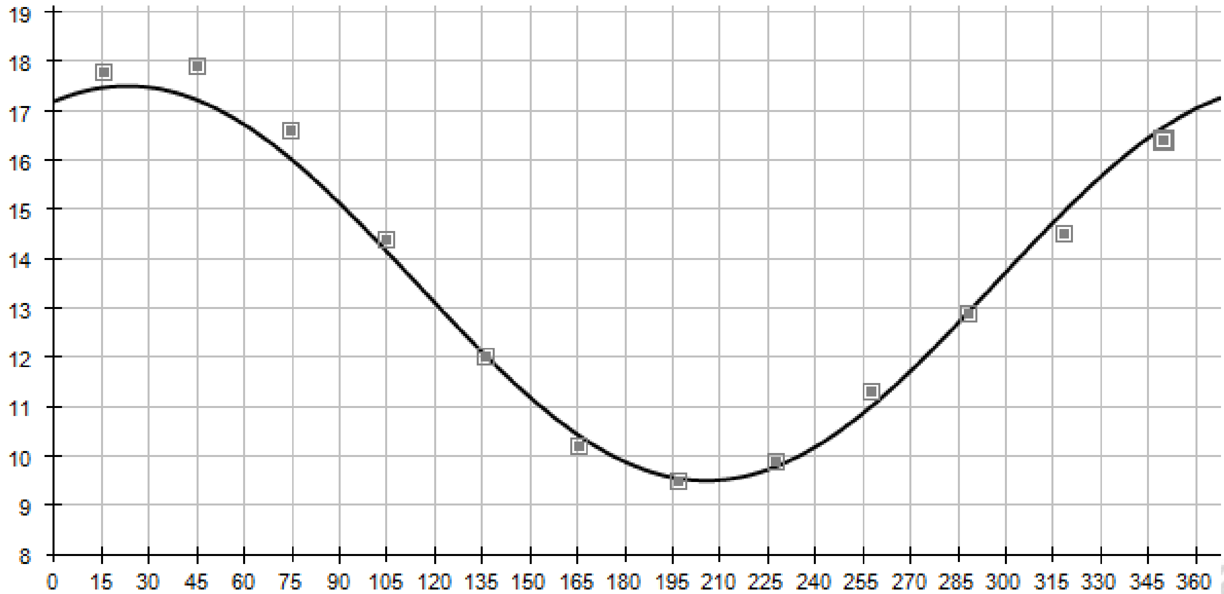
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