Practice for L3 Equations #6

1. Solve the system of equations

   $x + 2y + 2z = 1$ $\quad$ $3x + 4y + 2z = 2$ $\quad$ $2x + 2y + 3z = 4$

2. A printing firm charges $63 for 5 bindings, 1000 pages and 5 inserts.

   It charges $268 for 20 bindings, 800 pages and 40 inserts.

   It charges $94 for 10 bindings and 1200 pages.

   How much would it charge for 2 bindings, 200 pages and 5 inserts?

3. Four entrées, four main courses and four desserts cost $280.

   Three entrées, three mains and two desserts cost $195.

   Three entrées cost the same as four desserts.

   How much is a dessert?

4. Peter picks three numbers that add up to 46.

   The middle number is four-fifths of the difference between the largest and smallest.

   The largest number is four more than the other two numbers added together.

   What are the numbers?

5. Describe fully the nature of the system of equations below:

   $2x + y - z = 8$ $\quad$ $3x + 4y + z = 10$ $\quad$ $x - 2y - 3z = 6$

6. Consider the following system of equations:

   $x + 2y + kz = 6$ $\quad$ $x + y + z = 9$ $\quad$ $3x + 4y + 2z = 12$

   Find k so that the system cannot be solved and explain with a full geometrical description.

Answers: Practice for L3 Equations #6

1. **Solution: $x = 2$, $y = {}^{-}1.5$, $z = 1$**

2. $5b + 1000p + 5i = 63$ $\quad$ $20b + 800p + 40i = 268$ $\quad$ $10b + 1200p = 94$

   So $b = 8.2$, $p = 0.01$, $i = 2.4$. Which means $2b + 200p + 5i =$ **$48.40**

3. $4e + 4m + 4d = 280$ $\quad$ $3e + 3m + 2d = 195$ $\quad$ $3e + 0m - 4d = 0$

   Solving gives a dessert costs $15

4. $x + y + z = 46$ $\quad$ $x + y + z = 46$

   $y = {}^{4}/_{5}(z - x)$ $\quad$ $5y = 4(z - x)$ $\quad$ $4x + 5y - 4z = 0$

   $x + y + 4 = z$ $\quad$ $x + y - z = -4$

   Which makes the four numbers **5, 16 and 25**

5. ① $2x + y - z = 8$ $\quad$ ② $3x + 4y + z = 10$ $\quad$ ③ $x - 2y - 3z = 6$

   taking 2① – 1② – 1③ gives the equation: $0 = 0$ so the system is **dependent**.

   There are an **infinite number of solutions**. All three **planes** mutually **intersect along a common line**.

6. ① $x + 2y + kz = 6$ $\quad$ ② $x + y + z = 9$ $\quad$ ③ $3x + 4y + 2z = 12$

   taking 1① + 2② – 1③ cancels out the x and y components. To do the same for the z component gives that $\boldsymbol{k = 0}$. Now taking the equations 1① + 2② – 1③ gives $0 = 12$

   This yields an inconsistent system, where there are **no solutions**. Taking each pair of planes and finding their line of intersection gives a system of **three parallel lines**.