Y13 Extension – 3 × 3 Equations

1. If I have the equation $4x + 3y + z = 12$. What does that look like as a physical representation in 3 dimensions?

2. Find k so that the system of equations below is consistent:
   $9x + y + 3z = -3$ $\quad$ $x + 6y + 2z = 6$ $\quad$ $15x + 24 = ky$

3. Write the solutions for a and b in terms of c for the following system of equations:
   $2a + \frac{b}{2} + \frac{c}{12} = 2$ $\quad$ $a - 3b + \frac{c}{3} = 5$ $\quad$ $-6a - 60b + 5c = 66$

4. Find the equation of the parabola that passes through (2, 3.23), (3, 5.33) and (5, 8.33).

5. As much as possible solve the following system of equations :
   $a + b + c + d = 28$ $\quad$ $2a - b + 2c + d = 14$ $\quad$ $a - b + c + 2d = 15$

6. Find the integer solutions to the system of equations:
   $7x + 2z = 20 + y$ $\quad$ $x + y + z = 10$ $\quad$ $7y + 4z = 40 + x$

Answers: Y13 Extension – 3 × 3 Equations

1. If we set x, y and z in turn to various values we can locate points on the plane. The easiest to visualise is the points on the axes.

   A plane, passing through (0, 0, 12), (0, 4, 0) and (3, 0 , 0),

   A plane, perpendicular to the vector $\begin{pmatrix}4\\3\\1\end{pmatrix}$ passing through (1, 1, 5)

2. Rewrite as: ① $9x + y + 3z = -3$ ② $x + 6y + 2z = 6$ ③ $15x - ky = -24$

   Solving $9a + 1b = 15$ and $3a + -2b = 0$ gives us $a = 2$ and $b = -3$

   This gives us our multiples, so that eqn ③ = 2 × eqn ① + −3 × eqn ②

   Using our multiples, $-k = 2 \times 1 + -3 \times 6 = -16$, so $\boldsymbol{k = 16}$

3. Multiply the equations out to get rid of fractions, giving us:

   ① $24a + 6b + c = 24$ ② $3a - 9b + c = 15$ ③ $-6a - 60b + 5c = 66$

   to get rid of b 3×① + 2×② ⇒ $78a + 5c = 102$ ⇒ ④ $c = 20.4 - 15.6a$

   to get rid of a −1×① + 8×② ⇒ $-78b + 7c = 96$ ⇒ ⑤ $c = \frac{96}{7} + \frac{78}{7}b$

   Changing the subject for ④ and ⑤ gives $\boldsymbol{a = \frac{20.4 - c}{15.6}}$ **and** $\boldsymbol{b = \frac{7c - 96}{78}}$

4. A parabola is $a x^2 + b x + c = d$ which we can use to write equations

   $4a + 2b + c = 3.23$, $9a + 3b + c = 5.33$ and $25a + 5b + c = 8.33$

   Which solves on the calculator to give: $\boldsymbol{y = 0.2x^2 + 3.1x + 2.17}$

5.

$a + b + c + d = 28$	$(a + c) + b + d = 28$	$k + b + d = 28$
$2a - b + 2c + d = 14$	$2(a + c) - b + d = 14$	$2k - b + d = 14$
$a - b + c + 2d = 15$	$(a + c) - b + 2d = 15$	$k - b + 2d = 15$

   k = ***a* + *c* = 8, *b* = 11, *d* = 9** **No solution is possible for *a* and *c* separately**

6. ① $7x - y + 2z = 20$ ② $x + y + z = 10$ ③ $-x + 7y + 4z = 40$

   to get rid of x ① − 7×② ⇒ $-8y - 5z = -50$ ⇒ ④ $8y + 5z = 50$

   to get rid of y ① + ② ⇒ ⑤ $8x + 3z = 30$

   to get rid of z ① − 2×② ⇒ $5x - 3y = 0$ ⇒ ⑥ $5x = 3y$

   Eqn ⑥ is the useful one, as it has the least terms. It limits us (for integer solutions) to situations where we are dealing with a multiple of 15 because otherwise $5x \neq 3y$.

   We can generalise that to the situation where $x = 3n$ and $y = 5n$ for n any integer. We can then substitute this into eqn ② so that $z = 10 - 3n - 5n$.

   The solutions follow the scheme: $\boldsymbol{x = 3n, y = 5n, z = 10 - 8n \quad \forall n \in \mathbb{Z}}$