

Practice for L3 Probability #6

Question One

In a group of 40 students (24 boys and 16 girls), 15 of the boys and 12 of the girls study Physics.

- Two students are selected at random from the group. Find the probability that both these students are girls.
- A boy and a girl are selected at random from the group. Find the probability that only one of the two students studies Physics.
- Find the probability that a randomly selected student does not study Physics, given that he is a boy.
- Are the events "a randomly selected student is a boy" and "a randomly selected student studies Physics" independent? *You must justify your answer.*
- 60% of the boys who study Physics also study Chemistry. 75% of the girls who study Physics also study Chemistry.

Find the probability that a randomly selected student studies both Physics and Chemistry.

Question Two

A student sits three Physics external achievement standards.

- The first is worth four credits and has a historical pass rate of 60%.
- The second is worth three credits and has a historical pass rate of 80%.
- The third is worth three credits and has a historical pass rate of 75%.

Find the expected value of credits passed.

Question Three

A boat has a selection of twelve different flags.

- How many different variations can it have flying if it can fit three on the mast and their order is important?
- How many different variations can it have flying if it can fit three on the mast and their order is not important?

Question Four

At a tuck-shop 55% of the students buy a pie, 40% buy a filled roll and 10% buy both.

- What percentage buy neither a pie nor a filled roll?
- What is the probability that a person buying a pie will also buy a filled roll?

Answers: Practice for L3 Probability #6

1. a) $\frac{16}{40} \times \frac{15}{39} = 0.1538$ (or $\frac{{}^{16}C_2}{{}^{40}C_2}$)

b)

{	Boy doing Physics	{	Girls doing Physics	$15/25 \times 12/16$
		{	Girl not doing Physics	$15/25 \times 4/16$
{	Boy not doing Physics	{	Girl doing Physics	$10/25 \times 12/16$
		{	Girl not doing Physics	$10/25 \times 4/16$

$P(\text{only one doing Physics}) = 15/25 \times 4/16 + 10/25 \times 12/16 = 0.45 (= \frac{9}{20})$

c) $P(\text{Phy}' | \text{Boy}) = \frac{9}{24} = 0.375$ (Contingency table might help: of gender vs physics)

d) $P(\text{Boy}) \times P(\text{Physics}) = \frac{24}{40} \times \frac{27}{40} = 0.405 \neq P(\text{Boy} \cap \text{Physics}) = \frac{15}{40} = 0.375$
So they are **not** independent.

e) 15 boys do physics, so $0.6 \times 15 = 9$ boys do both physics and chemistry.
12 girls do physics, so $0.75 \times 12 = 9$ girls do both physics and chemistry.
18 out of 40 students study both, so $P(\text{study both}) = \frac{18}{40} = 0.45$

2. Expected value for independent events can be added:

$$E(A+B+C) = E(A)+E(B)+E(C)$$

$$E(\) = \text{pass rate} \times \text{credits} + \text{fail rate} \times 0$$

$$E(\text{credits}) = 4 \times 0.6 + 3 \times 0.8 + 3 \times 0.75 = 7.05$$

3. a) ${}^{12}P_3 = 12 \times 11 \times 10 = 1,320$

$${}^{12}C_3 = \frac{12!}{3!9!} = 220$$

4.

	Roll	No roll	
Pie	10	45	55
No pie	30	15	45
	40	60	100

a) 15% buy both.

b) $P(\text{Roll} | \text{Pie}) = \frac{10}{55} = 0.182$