You are advised to spend 60 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

(a) Write $\frac{5}{3+\sqrt{2}}$ in the form $a+b\sqrt{2}$ where a and b are rational numbers.

(b) What is the remainder when x^4-5x^2-4x+8 is divided by $x-3$?

(c) Solve the following equation for x in terms of m:

$$\sqrt{x-m}=\sqrt{x}-5$$

Assessor's use only

(d) Write $\dfrac{\sqrt{18k}\left(\sqrt{8k-2k}\right)}{\sqrt{54}-\sqrt{24}}$ in its simplest exact form.

(e) The equation $ax^2 + bx + c = 0$ has two roots.

Prove that if one root is twice the other then $2b^2 = 9ac$.

QUESTION TWO

Assessor's use only

(a) If $u = 6B\text{cis}\frac{\pi}{3}$ and $v = 2B\text{cis}\frac{\pi}{4}$, write $\frac{u}{v}$ in polar form.

(b) Solve $x^2 - 12x + 41 = 0$

Express the solutions in the form $x = a + \sqrt{b}\text{i}$, where a and b are rational numbers.

(c) Find all the solutions of the equation $z^4 = n\text{i}$, where n is a positive real number.

Write your solutions in polar form as expressions in terms of n.

(d) One root of the equation $z^3 - 4z^2 - 2z + n = 0$ is $z = 3 + i$.

Find the value of the real number *n* and the two other roots.

Assessor's use only

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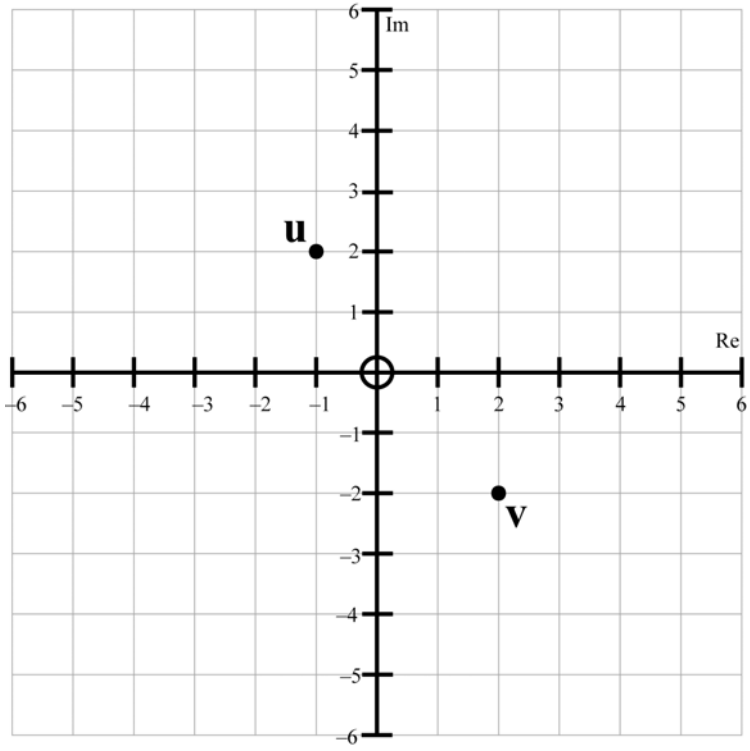
(e) Find the complex number $z = x + \mathrm{i}y$ that satisfies the equation $\frac{1}{z} + \frac{3}{\overline{z}} = 2 + \mathrm{i}$.

QUESTION THREE

Assessor's use only

(a) Complex numbers u and v are plotted on the Argand diagram.
Plot uv on the Argand diagram below.



(b) If $u = 4 + ki$ and $v = 2 + ki$, find the value of k if $u \cdot v = -17 + 30i$.

(c) If $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b}$, prove that $x + y + z = 0$.

(d) If $z = 1 + 3\text{i}$ and $u = \frac{1}{z} + \text{i}$, find the modulus of u.

Assessor's use only

(e) Solve the following equation for x.

$$\frac{x-1}{t-1}-\frac{x+1}{2x}+\frac{1}{x}=0$$

One	Expected Coverage	Achievement	Merit	Excellence
(a)	$\frac{15}{7}-\frac{5}{7}\sqrt{2}$	Correct solution.		
(b)	32	Correct solution.		
(c)	$x-m=x-10\sqrt{x}+25$ $10\sqrt{x}=m+25$ $100x=(m+25)^2$ $x=\frac{(m+25)^2}{100}$	Correct expression for $\sqrt{x}$.	Correct solution.	
(d)	$=\frac{\sqrt{144k^2}-\sqrt{36k^2}}{3\sqrt{6}-2\sqrt{6}}$ $=\frac{12k-6k}{\sqrt{6}}$ $=\frac{6k}{\sqrt{6}}$ $=\sqrt{6}k$	Correct expression with irrational denominator.	Correct solution.	
(e)	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x_1=2x_2\Rightarrow$ $-b-\sqrt{b^2-4ac}=2\left(-b+\sqrt{b^2-4ac}\right)$ $b=3\sqrt{b^2-4ac}$ $b^2=9b^2-36ac$ $36ac=8b^2$ $9ac=2b^2$		Correct equation for relationship between *a, b, c.* NOTE: Can get to line $b=3\sqrt{b^2-4ac}$ by logic.	Correct answer with a logical chain of reasoning and correct mathematical statements.

Not Achieved	NØ	No response; no relevant evidence.
	N1	Candidate provides ONE correct relevant algebraic step towards a solution.
	N2	Candidate provides ONE correct solution.
Achievement	A3	Candidate provides TWO of the Achievement criteria.
	A4	Candidate provides THREE of the Achievement criteria.
Merit	M5	Candidate provides ONE of the Merit criteria.
	M6	Candidate provides TWO of the Merit criteria.
Excellence	E7	Candidate provides ALL of the Excellence criteria with minor errors ignored.
	E8	Candidate provides ALL of Excellence criteria correctly.

Two	Expected Coverage	Achievement	Merit	Excellence
(a)	$3\text{cis}\frac{\pi}{12}$	Correct solution.		

(b)	$x = 6 \pm \sqrt{5}\text{i}$	Correct solution.		
(c)	$z^4 = n\text{cis}\dfrac{\pi}{2}$ $z_1 = \sqrt[4]{n}\text{cis}\dfrac{\pi}{8}$ $z_2 = \sqrt[4]{n}\text{cis}\dfrac{5\pi}{8}$ $z_3 = \sqrt[4]{n}\text{cis}\dfrac{-3\pi}{8}$ $z_4 = \sqrt[4]{n}\text{cis}\dfrac{-7\pi}{8}$	One correct solution.	Four correct solutions. (Accept positive arguments.)	
(d)	$(z-3+\text{i})(z-3-\text{i})$ $= z^2 - 6z + 10$ $(z^2 - 6z + 10)(z-a)$ $= z^3 - (6+a)z^2 + (10+6a)z - 10a$ $= z^3 - 4z^2 - 2z + n$ $a = -2$ $n = 20$ Other roots are $z = -2$ and $z = 3 - \text{i}$	Correct value of either n OR other roots.	Correct value of n other AND roots.	
(e)	$\dfrac{1}{x+\text{i}y} + \dfrac{3}{x-\text{i}y} = 2+\text{i}$ $\dfrac{x-\text{i}y+3(x+\text{i}y)}{x^2+y^2} = 2+\text{i}$ $\dfrac{4x}{x^2+y^2} + \dfrac{2y\text{i}}{x^2+y^2} = 2+\text{i}$ $\dfrac{4x}{x^2+y^2} = 2$ $\dfrac{2y}{x^2+y^2} = 1$ $\Rightarrow x = y$ $\therefore \dfrac{4x}{2x^2} = 2$ $x = 1,\ y = 1$ $z = 1+\text{i}$		Correct expression with real denominator.	Correct answer with a logical chain of reasoning and correct mathematical statements.

Not Achieved	NØ	No response; no relevant evidence.
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	N2	Candidate provides ONE correct solution.
Achievement	A3	Candidate provides TWO of the Achievement criteria.
	A4	Candidate provides THREE of the Achievement criteria.

Merit	M5	Candidate provides ONE of the Merit criteria.
	M6	Candidate provides TWO of the Merit criteria.
Excellence	E7	Candidate provides ALL of Excellence criteria with minor errors ignored.
	E8	Candidate provides ALL of Excellence criteria correctly.

Three	Expected Coverage	Achievement	Merit	Excellence
(a)	$uv = 2 + 6\text{i}$	*uv* placed at correct point on Argand diagram.		
(b)	$u \cdot v = 8 + 4ki + 2ki + k^2 i^2$ $= (8 - k^2) + 6ki$ k = 5	Correct solution.		
(c)	$x = \frac{(b-c)y}{c-a}$ $z = \frac{(a-b)y}{c-a}$ $\therefore x + y + z =$ $\frac{(b-c)y}{c-a} + y + \frac{(a-b)y}{c-a}$ $= \frac{(b-c)y + (c-a)y + (a-b)y}{c-a}$ $= \frac{y(b-c+c-a+a-b)}{c-a}$ $= 0$	Correct expression of *x*, *y* or *z* in terms of one other variable.	Correct solution.	
(d)	$u = \frac{1}{1+3\text{i}} + \text{i}$ $= \frac{1-3\text{i}}{10} + \text{i}$ $= \frac{1}{10} + \frac{7}{10}\text{i}$ $\lvert u \rvert = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{7}{10}\right)^2}$ = $\sqrt{0.5}$ or $\frac{1}{\sqrt{2}}$	Correct expression for *u* with a rational denominator. Accept $0.1 + 0.7\text{i}$	Correct solution.	
(e)	$\frac{x-1}{t-1} - \frac{x+1}{2x} + \frac{2}{2x} = 0$ $\frac{x-1}{t-1} - \frac{x-1}{2x} = 0$ $\frac{x-1}{t-1} = \frac{x-1}{2x}$ $2x(x-1) = (x-1)(t-1)$ $2x(x-1) - (x-1)(t-1) = 0$ $(x-1)(2x-(t-1)) = 0$ $x = 1$ or $x = \frac{t-1}{2}$		Correct equation with denominator of 1. (eg, line 4.)	Correct answer with a logical chain of reasoning and correct mathematical statements.

Not Achieved	NØ	No response; no relevant evidence.
	N1	Candidate provdes ONE correct relevant algebraic step towards a solution.
	N2	Candidate provides ONE correct solution.
Achievement	A3	Candidate provides TWO of the Achievement criteria.
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