Year 10 Number Theory

Introduction

"Number" is just the modern term for what would have been called Arithmetic in the past. That is the Maths associated with calculations with numbers, particularly percentages and proportions.

Language

Students need to take some time to learn all the terms associated with numbers. For many students failure to take the time to learn the language severely affects their ability to understand what are otherwise quite simple questions.

BEDMAS

The order of operations is important, particularly as students move into Algebra.

B = Brackets. These are done first.

$10 - (2 + 7) = 10 - 9 = 1$ whereas: $10 - 2 + 7 = 17 - 2 = 15$

E = Exponents (the powers of the numbers). They are done after brackets but before all else.

$8 + 6^2 = 8 + 36 = 44$ whereas $(8 + 6)^2 = 14^2 = 196$

DM = Division and Multiplication. They go before addition and subtraction.

$3 + 5 \times 8 = 3 + 40 = 43$

$4 \times 7 - 6 \div 3 = 28 - 2 = 26$

AS = Addition and Subtraction. They are done last.

A subtraction sign only applies to the term immediately following it.

$5 - 3 + 2 = 4$ $5 - 3 + 2$ does **not** mean $5 - (3 + 2) = 0$

The line of a square root sign or fraction indicates effectively that the items are bracketed. This is particularly important to remember when using a calculator.

$\sqrt{4+21}$	$= \sqrt{}(4 + 21)$	$= \sqrt{25}$	$= 5$	(not $\sqrt{4} + 21 = 23$)
$\frac{4}{2+2}$	$= \frac{4}{(2+2)}$	$= \frac{4}{4}$	$= 1$	(not $4 \div 2 + 2 = 4$)
$\frac{4+8}{2}$	$= \frac{(4+8)}{2}$	$= \frac{12}{2}$	$= 6$	(not $4 + 8 \div 2 = 8$)

Blue Beta:
New Beta:

Rounding

Decimal Places: the number is chopped off at the specified number of decimal places (d.p.). Then if the following digit is 5, 6, 7, 8 or 9 then the last digit is increased by 1.

6.8463 to 2 d.p. = 6.84|63 = 6.85   rounded up because the next digit is a 6

7.3541 to 2 d.p. = 7.35|49 = 7.35   not rounded up, because the next digit is a 4

Significant Figures: the number is chopped off counting the required number of significant figures (s.f.) which are any digit that are not merely place-holders. Then if the following digit is 5, 6, 7, 8 or 9 then the last digit is increased by 1.

68463 to 3 s.f. = 684|63 = 68500   rounded up because the next digit is a 6

0.007632 to 2 s.f. = 0.0076|32 = 0.0076   not rounded up, because the next digit is a 4

The answer must be given to the correct number of decimal places or significant figures, so we still write any zeros at the end of the number.

7.0033 to 2 d.p. = 7.00|33 = 7.00   (not 7, even though that is the same value)

8.403 to 3 s.f. = 8.40|3 = 8.40   (not 8.4, even though that is the same value)

Only the digit immediately after the cut-off point is considered. It does not matter what follows that, even if it is a string of 9s.

54.84999 to 1 d.p. = 54.8|4999 = 54.8

If the rounded up digit is a 9, then the rounding is carried up to the next digit.

4.97 to 1 d.p. = 4.9|7 = 5.0

597 to 2 s.f. = 59|7 = 600

Any zero is a significant figure except leading ones.

202,999 to 2 s.f. = 20|2999 = 200,000   (the 0 after the 2 is significant)

0.03055 to 3 s.f. = 0.0305|5 = 0.0306   (the first zeros are only significant **after** the 3)

Students should be aware that the purpose of rounding is to remove unnecessary or inaccurate precision, but that the result must be a similar sized answer.

67912 to 2 s.f. = 68000, not 68 – the final three zeros are required as place holders.

Students should always round to a sensible number of decimal places, even when the question does not specify the number. “Sensible” depends a bit on the context, but it is rarely reasonable to give answers to more than a couple of decimal places.

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Standard Form

Also known as Scientific Notation, it is mostly used for very large or very small numbers.

The number is rewritten as a base and an exponent: $a \times 10^{b}$.

The base is found by taking the number and putting the decimal point after the first significant figure. (This means that the base will be greater than 1, but less than 10.)

The exponent will be the number of places the decimal point had to move to find the base.

Students are expected to be able to work both to and from scientific notation.

If a number has no decimal point written, then it is at the end (because there is no decimal part of the number).

$7433 = 7.433 \times 10^{3}$

The exponent will be negative if the number is less than 1, as the decimal point is being moved in the opposite direction. This does not make the number negative.

$0.000452 = 4.54 \times 10^{-4}$

Scientific notation can be checked for numbers that aren't too big or small by entering it as written into the calculator.

If you type "5.87 × 10 ^ 3" into your calculator, you get 5870.

Calculators will use scientific form whenever the answer is very large or very small, even if set to use normal form. You should recognise the way your calculator represents this, as different ones do it differently.

Casio scientifics write scientific notation with a very small "× 10"

Casio graphics write it with an **E** before the exponent part: 4.5 **E** +3 = 4.5×10^{3}

Negative Numbers

Although students can mostly get by at lower levels using a calculator, they are **severely** disadvantaged when they attempt Algebra if they have not mastered negatives.

A negative results when a larger number is taken away from a smaller number.

$10 - 12 = {}^{-}2$

Subtracting a negative is the same as adding a positive.

$8 - {}^{-}6 = 8 + 6$

When we multiply or divide with negatives, we calculate the number answer like normal. Afterwards, we add up all the negatives, and cancel out any pairs.

${}^{-}5 \times 8 = {}^{-}40$

${}^{-}5 \times {}^{-}8 = {}^{-\,-}40 = {}^{+}40$

${}^{-}5 \times {}^{-}8 \div {}^{-}2 = {}^{-\,-\,-}20 = {}^{-}20$

The phrase "two negatives make a positive" should be avoided, as it confuses students adding two negative numbers. It is much better to say "**two side-by-side negatives cancel**" or similar. Two negative answers added together give a negative.

${}^{-}7 + {}^{-}8 = {}^{-}15$

Adding a negative is the same thing as subtracting the positive. There is no difference between a "minus" and a "negative", which is why the same sign is used.

$5 - 3 = 5 + {}^{-}3$

We can reorder a sum with addition (but not subtraction). One way to simplify sums with negatives or subtractions is to use the ability to reorganise additions into any order.

${}^{-}3 + 5 \quad = 5 + {}^{-}3 \quad = 5 - 3 \quad = 2$

$7 - 13 + 8 \quad = 7 + {}^{-}13 + 8 \quad = 7 + 8 + {}^{-}13 \quad = 15 - 13 \quad = 2$

Blue Beta:
New Beta: