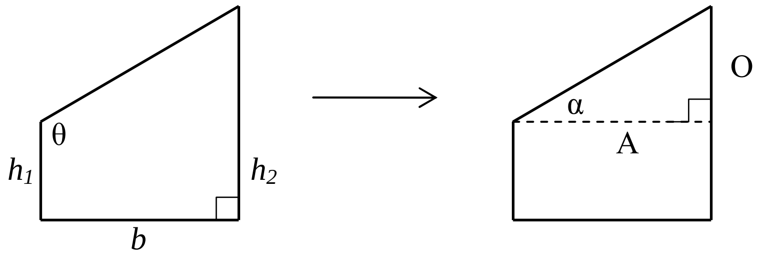
Using Geometry when the Triangle is not 90°

There are a fairly limited number of ways that Merit and Excellence questions can be posed, and students hoping for a good mark should be able to memorise all the possible options, more or less.

> The trigonometric relationships given by $S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$ only work for right angle triangles.
>
> To find the triangle often involves a little geometry.

A trapezium can be converted to rectangle and triangle.

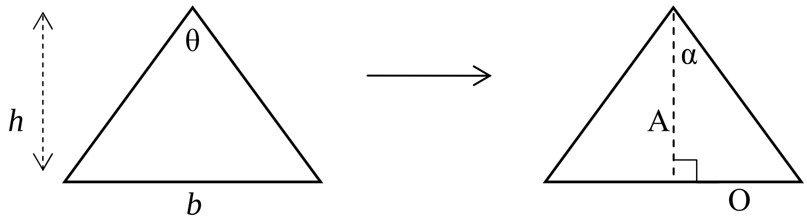


$\alpha = \theta - 90°$

$O = h_2 - h_1$

$A = b$

An isosceles triangle can be converted into two identical right angle triangles.

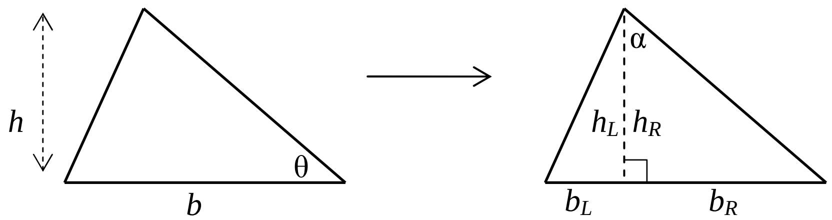


$\alpha = \frac{1}{2}\theta$

$O = \frac{1}{2} b$

$A = h$

Other triangles can be converted into two non-identical right angle triangles, who share a side.

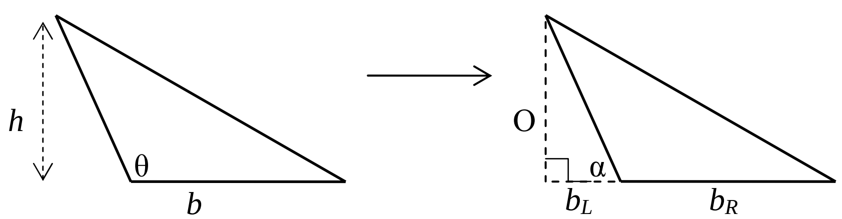


$\alpha = 90° - \theta$

$b_L + b_R = b$

$h_L = h_R = h$

This includes the situation where the original is obtuse, and the second triangle is **outside** the first.



$\alpha = 180° - \theta$

$b_R = b$

$O = h$

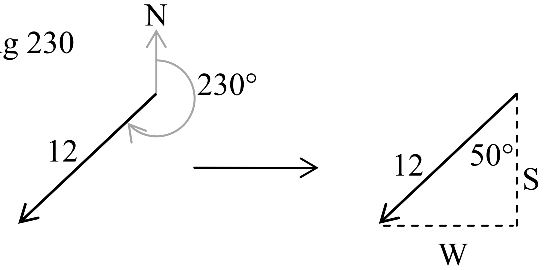
Using Geometry for Directions and Bearings

Merit and Excellence questions often involve directions.

> A bearing is the angle clockwise from North. They are written in three digits, with no degree sign.
>
> Any direction can be broken up into components in a North-South and East-West pair.

When a bearing is greater than 090 the triangle needs to be generated from S, W or E so that it is right angled.

If I go 12 km at a bearing 230

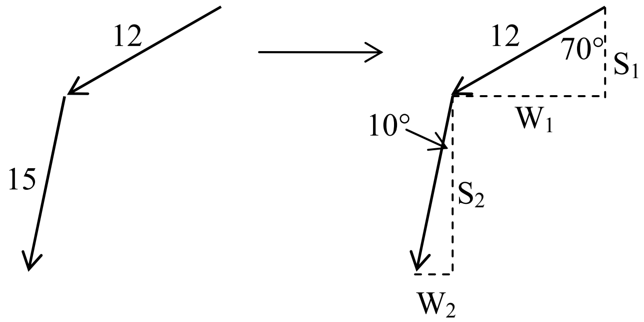


$S = \cos 50° \times 12$

$W = \sin 50° \times 12$

When adding two paths, we need to do this via their NS and EW components, which we put together at the end.

If I go 12 km at bearing 250, then 15 km at bearing 190.



The distance West of the starting point is: $W_1 + W_2 = \sin 70° \times 12 + \sin 10° \times 15$

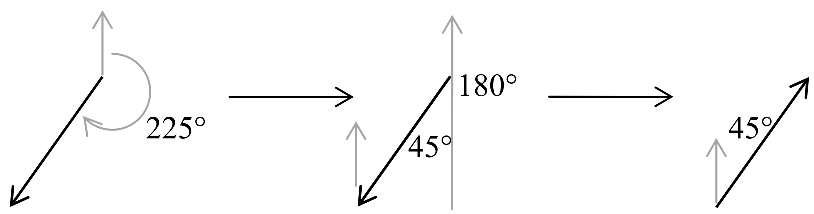
The distance South of the starting point is: $S_1 + S_2 = \cos 70° \times 12 + \cos 10° \times 15$

Overall distance is found using Pythagoras: $\text{Distance}^2 = (W_1 + W_2)^2 + (S_1 + S_2)^2$

Overall bearing taken is found using Trigonometry: $\theta = \tan^{-1}\left(\frac{W_1 + W_2}{S_1 + S_2}\right) + 180°$

Many bearing questions require the return bearing be calculated. These can be answered by parallel line geometry, on the basis that all Norths are parallel.

If I go out at bearing 225, then to return I must go at 045.



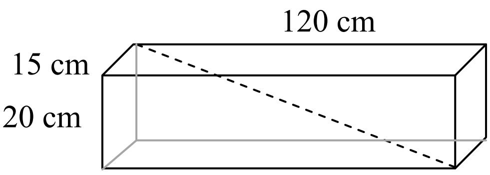
Questions in Three Dimensions

Pythagoras Theorem in 3D is $H^2 = a^2 + b^2 + c^2$, where a, b and c, must be all at 90° to each other.

When dealing with the angle of a line to a plane, dropping directly down from the point is required.

The key to these is understanding the geometry, especially since any picture can only show the situation in two dimensions. Students need to take care to understand the situation being described fully before attempting to answer it.

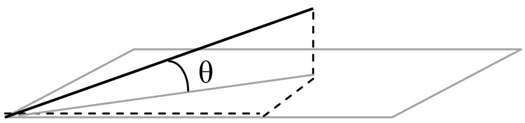
The longest diagonal distance across a box is the standard 3D Pythagoras question. Each side is already at 90° to the others ("orthogonal") so the formula can be used directly.



The dotted distance is
$= \sqrt{120^2 + 20^2 + 15^2} = 122.6$ cm

If the angle between a line and a plane is to be found, the "shadow" of the line on the plane (as if from a directly overhead light) is found first, and the angle calculated from that.

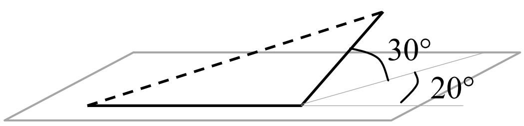
A line goes 4 in the *x* dimension, 3 in the *y* dimension, and 2 in the *z* dimension. Find θ.



The length of the "shadow" on the *x*–*y* plane is = $\sqrt{4^2 + 3^2} = 5$. So the angle formed with the plane, $\theta = \tan^{-1}\left(\frac{O}{A}\right) = \tan^{-1}\left(\frac{2}{5}\right) = 21.8°$

In the reverse situation, where the angle is given but only some of the dimensions, the process is again to work via the "shadow" on the plane given, generally breaking it into components.

A line goes along a plane 10 cm, then rises out of the plane 8 cm, at 30° to the plane and 20° to the first line. Find the dotted distance shown.

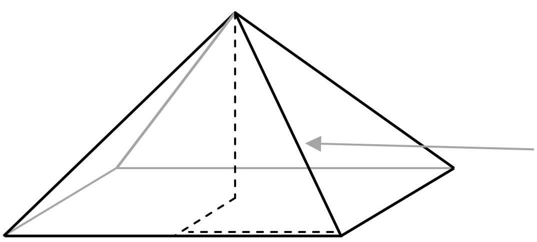


The second line has a vertical height from the plane = sin30° × 8 = 4 cm and a shadow at 90° to that **and** the first line = cos30° × 8 = 6.93 cm.

Dotted line = $\sqrt{10^2 + 4^2 + 6.93^2} = 12.8$cm

When working with pyramids the key geometric point is that each face is isosceles, so that halving it gets you to the midpoint (either across or deep, or both).

A pyramid has a base of 8 cm, and a height of 6 cm. Find the length of each rising edge.



The centre of the base is 4 cm along and 4 cm in from the edge. Using 3D Pythagoras, since we now have all the lengths at 90° to each other:

Length = $\sqrt{4^2 + 4^2 + 6^2}$
= 8.25 cm