

Sheet #22a

Simplify so that all the powers are positive (this may involve fractions):

1) $(x^4)^{-2}$

2) $\frac{y^6}{y^{-2}}$

3) $(xy^4)^2$

4) $\left(\frac{5}{a^3}\right)^{-2}$

5) $\frac{5y^4}{10y^{-1}}$

6) $\frac{p^{-2}}{p^{-6}}$

7) $\left(\frac{a}{b^{-2}}\right)^3$

8) $5(2y^{-2})^3$

Simplify to positive numerical exponents:

9) $\frac{\sqrt{x}}{x}$

10) $s(s^3)^{0.5}$

11) $\frac{\sqrt{x^9}}{\sqrt{x}}$

12) $\sqrt[2]{t} \times \sqrt[3]{t}$

13) $\sqrt{\frac{25x^9}{16y^{16}}}$

14) $6(ab^{-1})^2$

15) $\frac{y^6}{2} \div y^{-2}$

16) $(5ab^{-2})^{-2}$

Answers : Sheet #22a

The steps for many of these can be done in different orders (e.g. flipping for the negative, then squaring is the same as squaring then flipping for the negative).

$$1) \quad (x^4)^{-2} = \frac{1}{(x^4)^2} = \frac{1}{x^8} \quad \text{or} \quad (x^4)^{-2} = (x^4)^{2 \times -1} = (x^4 \times x^4)^{-1} = (x^8)^{-1} = \frac{1}{x^8}$$

$$2) \quad \frac{y^6}{y^{-2}} = \frac{y^6 \times y^2}{1} = \frac{y^8}{1} = y^8 \quad \text{Because} \quad \frac{1}{x^{-1}} = x$$

$$3) \quad (xy^{-3})^2 = xy^{-3} \times xy^{-3} = x^2y^{-6} = \frac{x^2}{y^6}$$

$$4) \quad \left(\frac{5}{a^3}\right)^{-2} = \left(\frac{a^3}{5}\right)^2 = \frac{a^3}{5} \times \frac{a^3}{5} = \frac{a^6}{25} \quad \text{Because} \quad \left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$$

$$5) \quad \frac{5y^4}{10y^{-1}} = \frac{5y^4 \times y}{10} = \frac{\cancel{5} \times y^4 \times y}{\cancel{5} \times 2} = \frac{y^5}{2} \quad \text{The}^{-1} \text{ is only on the } y$$

$$6) \quad \frac{p^{-2}}{p^{-6}} = \frac{\cancel{p^{-2}}}{\cancel{p^{-2}} \times p^{-4}} = \frac{1}{p^{-4}} = p^4 \quad \text{or, if we flip first:} \quad \frac{p^{-2}}{p^{-6}} = \frac{p^{-2}}{p^{-6}} = \frac{p^6}{p^2} = p^4$$

$$7) \quad \left(\frac{a}{b^{-2}}\right)^3 = (ab^2)^3 = ab^2 \times ab^2 \times ab^2 = a^3b^6$$

$$8) \quad 5(2x^{-2})^3 = 5\left(\frac{2}{x^2}\right)^3 = 5 \times \frac{2}{x^2} \times \frac{2}{x^2} \times \frac{2}{x^2} = \frac{40}{x^6}$$

$$9) \quad \frac{\sqrt{x}}{x} = \frac{\sqrt{x} \times 1}{\sqrt{x} \times \sqrt{x}} = \frac{1}{\sqrt{x}} = \frac{1}{x^{0.5}} \quad (\text{or } \frac{1}{x^{1/2}}) \quad \text{Because } \sqrt{x} = x^{0.5}$$

$$10) \quad s(s^3)^{0.5} = s \times \sqrt{s^3} = s \times s^{1.5} = s^{2.5}$$

$$11) \quad \frac{\sqrt{x^9}}{\sqrt{x}} = \frac{x^{4.5}}{x^{0.5}} = \frac{x^4 \times \cancel{x^{0.5}}}{\cancel{x^{0.5}}} = x^4$$

$$12) \quad \sqrt[2]{t} \times \sqrt[3]{t} = x^{1/2} \times x^{1/3} = x^{5/6} \quad \text{Because} \quad \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$13) \quad \sqrt{\frac{25x^9}{16y^{16}}} = \frac{5x^{4.5}}{4y^8} \quad \text{Because} \quad \frac{5x^{4.5}}{4y^8} \times \frac{5x^{4.5}}{4y^8} = \frac{25x^9}{16y^{16}}$$

$$14) \quad 6(ab^{-1})^2 = 6 \times \frac{a}{b} \times \frac{a}{b} = \frac{6a^2}{b^2}$$

$$15) \quad \frac{y^6}{2} \div y^{-2} = \frac{y^6}{2} \div \frac{y^{-2}}{1} = \frac{y^6}{2} \times \frac{1}{y^{-2}} = \frac{y^6}{2} \times \frac{y^2}{1} = \frac{y^8}{2}$$

$$16) \quad (5ab^{-2})^{-2} = \left(\frac{5a}{b^2}\right)^{-2} = \left(\frac{b^2}{5a}\right)^2 = \frac{b^2}{5a} \times \frac{b^2}{5a} = \frac{b^4}{25a^2}$$