Year 11 Graphing Notes

Terminology

It is very important that students understand, and always use, the correct terms. Indeed, not understanding or using the correct terms is one of the main reasons students fail this topic.

The lines drawn with the scales (numbers) on the co-ordinate system are the **axes**.

The ***x*-axis** is horizontal: (“*x* is a cross) and the ***y*-axis** is vertical: (“*y* to the sky”)

A point on the system is written with co-ordinates in the form: **(*x* , *y*)**.

(2, 3) is a point, but 2, 3 is just some numbers and is not marked as a point, even if correct.

The ***x*-intercept** is where a line crosses the *x*-axis, the ***y*-intercept** where a line crosses the *y*-axis.

The point two lines cross at is their **intersection**.

A parabola turns at a **turning point** (which is either a **minimum** or a **maximum**, depending on whether it is a positive or negative parabola).

The **equation** of a line or curve will be written in the form *term = term*, generally in the form y =:

$y = 3x + 4$ is an equation of a line, but $3x + 4$ is not an equation and <u>will</u> be marked incorrect.

Discrete data is data that takes exact values:

e.g the number of people in a room, the number of tracks on an iPod, etc.

Continuous data is, at least potentially, is able to take any value:

e.g height, weight – and some data is almost continuous, such as salary

Other words that students must know (some will be introduced later in these notes):

gradient, **parallel**, **consecutive**, **straight**, **curve** and **translate**.

Graphing conventions

It may cost grades if you plot points that make no sense in the context of the question:

e.g. don’t plot values for negative time, for zero hours worked, or the cost of half a car.

You also need to follow these graphing conventions.

Dots vs Lines

If the *x*-values of a graph can only take on exact values (i.e. are discrete), then the points should be shown at a series of dots. They should not be connected by a line.

If is continuous so that *x* can take any value, or when it only takes discrete values but they are so close together than you cannot separate them, then a solid line is used.

Line Endings

If a graph is infinite it can be shown going off the grid, or should have an arrow on the end.

If a graph stops at a point it can either be drawn as a line or a line ending with a blob:

If a graph goes up to but does **not** include a point, the line ends with a open dot:

Drawing Lines and Curves

Drawing Lines Method #1 – Plotting Points

> A set of x values is put into the equation, and plotted against the resulting y values.
>
> A line should be shown as infinite, and must include exact x- and y- intercepts if there are any.

It pays to do the substitution of values using a table:

x	$3x - 4$	y	*gives point*
$^{-}2$	$3 \times {}^{-}2 - 4$	-10	$(^{-}2, {}^{-}10)$
2	$3 \times 2 - 4$	-2	(2, 2)
4	$3 \times 4 - 4$	8	(0, 8)

The plotted points must line up exactly.

The x values chosen don't matter, except that some are easier to do the maths with – choosing multiples to simplify fractional gradients is especially useful.

If a point calculated is off the co-ordinate grid you are given it must be discarded.

Using at least four widely spaced x values helps spot errors.

One of the points you should use to check is the y-intercept (the c in $y = mx + c$).

Drawing Lines Method #2 – Slope–Intercept

> Start with the y-intercept from the c value in the $y = mx + c$ form.
>
> Then plot points away from the axis using the gradient – for every 1 grid point across, you need to rise m up (or down, if negative).

You should not use this method unless you are comfortable with the relationship between a line and its equation. It is, however, a fast method, so suitable for Merit students.

Plotting one point by substitution helps spot errors.

Make sure to extend the graph up to any x-intercept.

Drawing Lines Method #3 – Cover-up

> Calculate the y value for an x value of 0, giving the y-intercept.
>
> Calculate the x value for a y value of 0, giving the x-intercept.
>
> Connect the two intercepts with a straight line.

While not generally very reliable for most students, this method can be very useful with awkward equations, such as $4y + 3x = 12$ or similar forms where there is a y-coefficient not equal to 1.

It is usually best to check that one other point fits the equation.

Horizontal and Vertical Lines

If the line is $y = c$, then the line will be horizontal line, passing through the y-axis at c.

If the line is $x = c$, then the line will be vertical line, passing through the x-axis at c.

One way of making sure you get the difference is to substitute values:

To plot $y = 4$, connect any two points where $y = 4$ such as (1, 4) and (5, 4).

To plot $x = {}^{-}3$, connect any two points where $x = {}^{-}3$ such as $({}^{-}3, 2)$ and $({}^{-}3, 5)$.

Make sure the line crosses an axis.

Drawing Parabolas

A set of x values is put into the equation, and plotted against the resulting y values.

The sketch must include the accurate points around the turning point of a parabola, and also any x and y intercepts. (It may take some time to find these if using trial and error.)

A table is helpful in tracking back any errors.

x	$(x - 3)(x + 1)$	y	gives point
3	0×4	0	(3, 0)
${}^{-}2$	${}^{-}5 \times {}^{-}1$	0	$({}^{-}2, 5)$
etc			

Using the factorised (bracketed) form of the quadratic as shown above is quicker and usually gives less mistakes.

If using the expanded $y = x^2$ form then extreme care is required with substituting negatives:

$({}^{-}2)^2 = 4$, not ${}^{-}4$.

You can usually tell if your error is with negatives as the curve will change to the left of the y-axis.

A parabola must be smooth and exactly symmetrical, so there is a mistake if it is not smooth.

If an $(x - a)$ is instead written as $(a - x)$ then that will reverse the parabola so that it goes down not up (like a leading negative sign). A bracket of type $(a - x)$ can be substituted as normal, just taking care to make any double negatives a positive.

Drawing the Curve

Parabolas are infinite, so do not stop them at your last calculated point, but indicate that they go on (arrows, dotted lines etc).

You are not marked on your ability to plot an exact parabola, so do not take too long plotting every point and drawing the curve exactly correct. Take care with the points around the turning point, and the intercepts – and connect them with a single, smooth curve – but don't plot every point precisely.

It is usually best to plot points and draw a trial curve in pencil, but the final curve should be in ink.

Equations of Lines

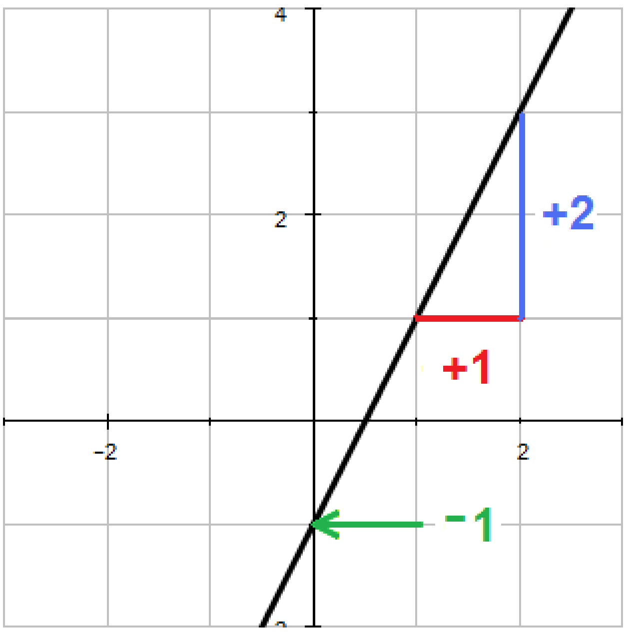
Write straight lines in the form $\boldsymbol{y = mx + c}$, where m is the gradient and c is the y-intercept.

Usually the easiest way to calculate gradient m is that it is how much the y changes for every one unit to the right in the x direct.

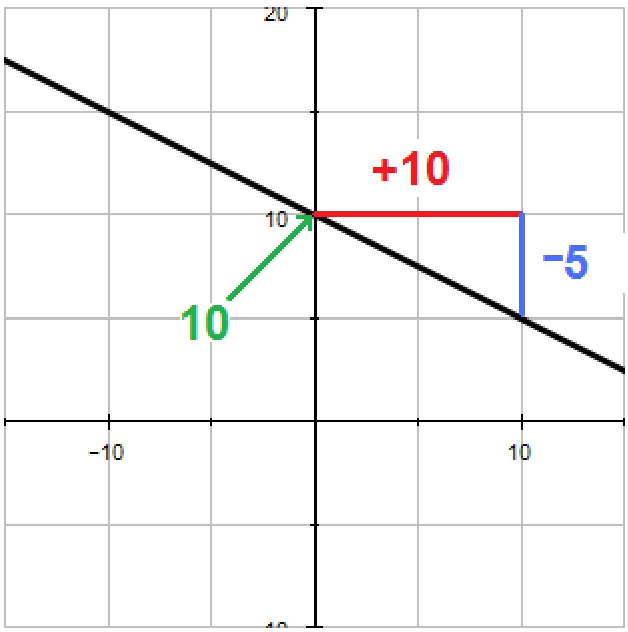
In more awkward situations you may need to use the formula

$$\text{Gradient} = \frac{rise}{run} = \text{the change in the } y \text{ values for a change in } x \text{ values} = \frac{change\ in\ y}{change\ in\ x}$$

The c is the value where the line crosses the y-intercept.



$m = {}^{+}2$, $c = {}^{-}1$ so $y = 2x - 1$



$m = {}^{-5}/_{10}$, $c = 10$ so $y = {}^{-}\,0.5x + 10$

It is vital to include the "y =" in your equation. Without it you will be marked incorrect, as you have an expression, not an equation.

Extreme care is required with negatives, especially with the gradients. Understanding what sort of value you expect for the gradient will help prevent errors.

i) the gradient is a larger number the nearer it gets to being vertical.

ii) if the line makes an angle less than 45° with the x-axis the gradient will be smaller than 1.

iii) if the slope heads down to the right, the gradient will be negative.

It is unimportant whether you write the negative as a minus sign or not.

$y = 3x - 5$ is the same as $y = 3x + {}^{-}5$

Horizontal and Vertical Lines

Write down any two points on the graph. If the x values are the same for both points then the graph is of the basic form $x = c$. If the y values are the same then the graph is of the form $y = c$.

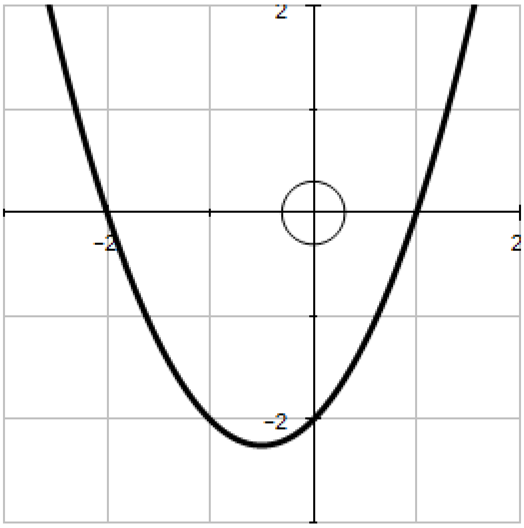
Equations of Parabolas

Parabola Equation Method #1 – Intercepts

This will generate an equation of the form: $y = k\,(x - i_1)(x - i_2)$

Find the two x-intercepts (i_1 and i_2) and put their **negatives** in the brackets after x.

Check with the y-intercept or turning point if the multiplier (k) is needed in front of the brackets.



Merit example:

The intercepts are at $x = 1$ and $x = {}^{-}2$.

Their negatives are $^{-}1$, and $^{+}2$.

So the equation is of the form:

$$y = (x - 1)(x + 2)$$

Putting in $x = 0$, gives the point $(0, {}^{-}2)$ so there is no need for a multiplier k.

The equation can also be written in different bracket order, or with + $^{-}1$

e.g. $y = (x + 2)(x + {}^{-}1)$ is the same equation

An intercept of (0, 0) gives one of the brackets at $(x - 0)$ which we just write as x.

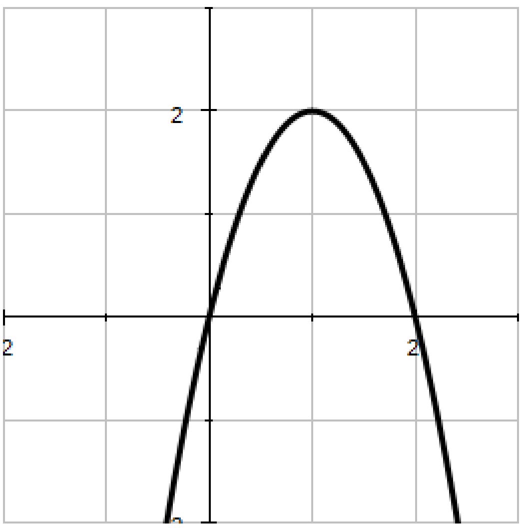
If the graph turns at the x-axis, then the intercepts are the same. We normally write these as squares

A graph that turns at (0, 3) is $y = (x - 3)(x - 3)$ but it is normally written as $y = (x - 3)^2$.

If the graph needs to be steeper than a large multiplier is required in front, and a small multiplier will make it flatter. A negative will make it go down, rather than up.

This method obviously requires you to have the intercepts, but you can expect that at Level 1.

If you have a graphics calculator, then checking the equation is an excellent idea.



Excellence example

The intercepts are at $x = 0$ and $x = 2$.

Their negatives are 0, and $^{-}2$.

So the equation is of the form:

$$y = (x - 0)(x - 2)$$

which we write

$$y = x\,(x - 2)$$

But the graph can be seen to be negative:

$$y = {}^{-}x\,(x - 2)$$

Putting in $x = 1$, gives the point (1, 1) which we don't have, so we need a multiplier k.

$$y = {}^{-}2x\,(x - 2)$$

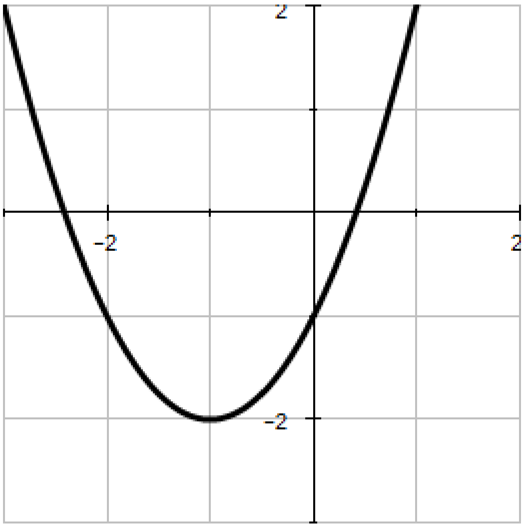
Parabola Equation Method #2 – Turning Point

This will generate an equation of the form: $y = k\,(x - t)^2 + c$

Find the x value of the turning point (t) and put its **negatives** in the brackets after x.

Find the distance the turning point is above or below the x-axis (c) and **add** that to the end.

Check with the y-intercept or turning point if the multiplier (k) is needed in front of the brackets.



Merit example:

The turning point is at $x = {}^{-}1$.

Its negatives is $x = {}^{+}1$.

So the equation is of the form:

$$y = (x + 1)^2 + c$$

The turning point is at $y = {}^{-}2$, so that is our c

$$y = (x + 1)^2 - 2$$

Putting in $x = 0$, gives the point $(0, {}^{-}1)$ so there is no need for a multiplier k.

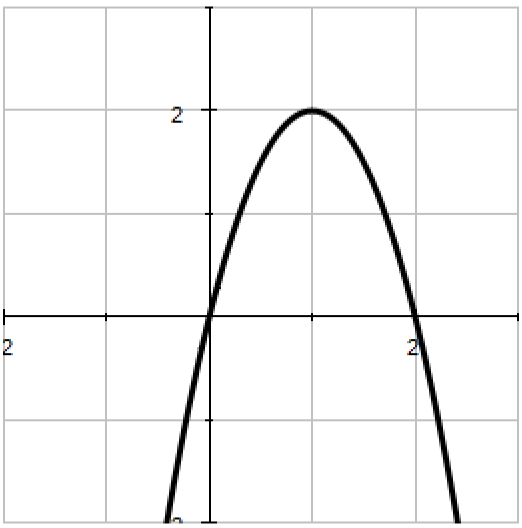
The equation can also be written as $y = (x + 1)^2 + {}^{-}2$

An turning point on the y-axis gives the brackets as $(x - 0)^2$ which we just write as x^2.

As with method 1, if the graph needs to be steeper than a large multiplier is required in front, and a small multiplier will make it flatter. A negative will make it go down, rather than up.

This method obviously requires you to have the turning point, which is normal at Level 1, but not always available. However it also allows you to write equations for parabolas that do not have intercepts.

If you have a graphics calculator, then checking the equation is an excellent idea.



Excellence example

The turning point is at $x = 1$.

Its negatives is $x = {}^{-}1$.

So the equation is of the form:

$$y = (x - 1)^2 + c$$

The turning point is at $y = 2$, so that is our c

$$y = (x - 1)^2 + 2$$

But the graph is clearly negative:

$$y = -\,(x - 1)^2 + 2$$

Putting in $x = 1$, gives the point $(1, 1)$ which we don't have, so we need a multiplier k.

$$y = {}^{-}2\,(x - 1)^2 + 2$$

Parabola Equation Method #3 – When Not Given a Co-ordinate System

> As you are not given the co-ordinate system, you can make your own.
>
> Generally it is easiest to work with the form: $y = k\,x\,(x - i_2)$, which you do by making one of your base points an intercept (usually the left one, because that leaves the parabola in the positives).
>
> The multiplier is then calculated from the turning point, which is in the middle of the intercepts.

Drawing a sketch is always a good idea and is the first thing you should do (and that means on paper, not just trying to do it in your head). It makes the questions a lot easier, and allows the marker to see what you are doing, so minor errors you make might still allow you the grade.

Understanding the symmetry of parabolas is an absolutely vital skill in answering word questions.

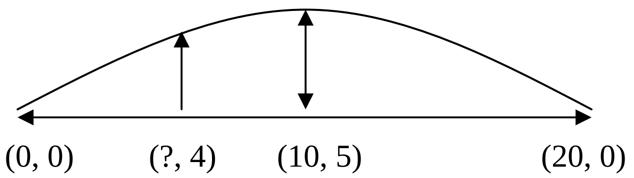
It can be helpful to reverse the picture if it goes the "wrong" way for how you want it.

If a diagram is given, it will usually have the variables some letters other than *x* and *y*. You can just cross them out and replace them with *x* and *y* if that suits you (which it usually will until you are much more comfortable with them).

Because parabolas have two *x* values for every *y* value – a going up value and a coming down value – many solutions to them involve quadratics with two solutions. Take care that you answer in the context of the question, which might mean discarding one of the answers.

Example 1

A ball is thrown in a parabola a distance of 20 metres, reaching a height of 5 metres at the top. When does the ball first reach a height of 4 metres off the ground?



We make the left point = (0, 0). Therefore the landing point is (20, 0) as it went 20 m.

We can use the intercept method to get an equation: $y = k\,x\,(x - 20)$

But we know that it is 5 m high at 10 m out (max is in centre) $\Rightarrow 5 = k \times 10 \times (10 - 20) \Rightarrow k = {}^{-}0.05$

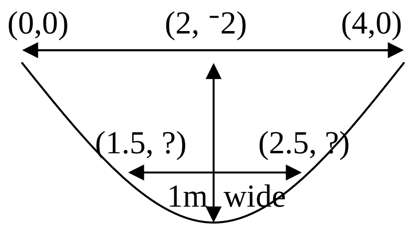
We are asked to find the *x* value at height = 4 $\Rightarrow$ $4 = {}^{-}0.05\,x\,(x - 20)$

Rearranging: $0 = {}^{-}0.05x^2 + x - 4$, which solves for $x = 5.53$ and 14.5. Only need first $\Rightarrow$ <u>5.53 metres</u>

Example 2

A ditch in the shape of a parabola is 4 metres wide and 2 metres deep.

How deep is it when it is 1 metre wide?



We make the left point = (0, 0). From width the other edge is (4, 0)

We can use the intercept method to get an equation: $y = k\,x\,(x - 4)$

But we know that it is 2 m deep at 1 m out (min is in centre)

$\Rightarrow {}^{-}2 = k \times 1 \times (1 - 4) \Rightarrow k = {}^{2}/_{3}$

We need to find the depth at $x = 1.5$ and 2.5 m, as they are the points when it is 1 m wide

$y = {}^{2}/_{3} \times 1.5 \times (1.5 - 4) =$ <u>2.5 metres deep</u>

Translating Graphs

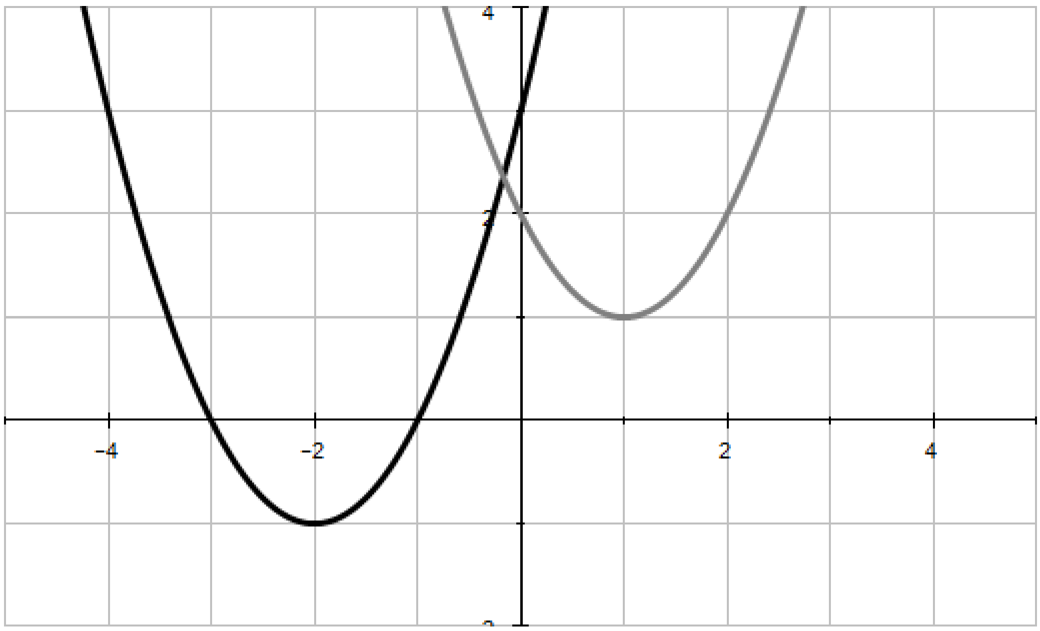
Translation is the process of shifting graphs so that their position changes but not their shape.

Move the previous graph in the x-direction first, either by finding the new intercepts when it is shifted, or finding the new turning point.

Then add the shift in the y-direction.

A translation may be given in vector form $\begin{pmatrix} x \text{ shift} \\ y \text{ shift} \end{pmatrix}$.

It is really important to do the x shift first and separately, and only then do the y shift.



To translate the black graph to the grey graph – a vector shift of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Intercept form – $y = k\,(x - i)(x - i)$

The black line is $y = (x + 3)(x + 1)$.

If you move it across three to the right, the intercepts go from $^{-}3$ and $^{-}1$ to 0 and 2 so the new form starts $y = x\,(x - 2)$.

It has been moved up two, so the grey curve is $y = x\,(x - 2) + 2$.

Turning point form – $y = k\,(x - t)^2 + c$

The black line is $y = (x + 2)^2 - 1$.

With the shift it becomes $y = (x - 1)^2 + 1$.

General form – $y = ax^2 + bx + c$ (not advised)

The graph is $y = x^2 + 4x + 3$

To shift it right $y = (x - 3)^2 + 4(x - 3) + 3$.

And then it is shifted up two so $y = (x - 3)^2 + 4(x - 3) + 1$

One advantage of using the turning point form is that it makes translations extremely easy.

If you are given only the general form, it is probably best to factorise it first to the intercept form.