Measurement Year 11

Rounding

Do not round early. Students should carry all decimal places in working until the **end** of their calculations. They should then give their answers sensibly rounded.

An answer is only accurate when it is properly calculated and has a real world meaning. Students must realise that a highly precise answer (many significant figures) is **not** better than a rounded one.

> If a circle has a diameter of 3 m then it has a circumference of $3 \times \pi$ but it is nonsensical to give this as 9.424777961 metres as this is to the nearest millionth of a millimetre!
>
> The number of people in NZ might be counted with total precision to 4,583,523. However as soon as this is counted it is inaccurate, since it is immediately wrong.

Temperature

Temperature is commonly measured in degree Celsius (°C).

There is also the scale used by scientists, the Kelvin scale (K), where $K = °C + 273$.

Students need to recognise some general points on the Celsius scale”

> 0°C is where ice forms from water
>
> 25°C is a pleasant day temperature
>
> 36–38°C is standard body temperature and 40°+ is a fever
>
> 100°C is where water boils

Temperature is a context where negative numbers are quite normally met in a real life. Students need to take care when finding the differences between temperatures when one is negative.

> – 4°C to +12°C is 16 degrees difference

MAN vs MATHS 2014

Units

In the whole topic of "Measurement" units are vital. **Every** answer must include the units used.

Convert **all** units at the **start** of any question, so that the whole calculation uses the measurements you want to work in.

Metric System

The Metric system has a set of standard base measurements:

Length	Metre	m
Mass (weight)	Gram	g

From these we have many multipliers, indicated by prefixes, but we only have to know three:

kilo-	k	× 1000
milli-	m	÷ 1000
centi-	c	÷ 100

There are a couple of related measurements in common use:

Tonne	= t	= 1,000 kg	
Hectare	= ha	= 10,000 m^2	(more usefully: 100m by 100m)
Litre	= L	= 1,000 cm^3	(more usefully: 1 mL = 1 cm^3)

Students need to be able to reliably convert between the various forms of units. Multiply or divide by the scale factor (and check that the answer is the right size afterwards).

× 1000

1 km = 1000 m

÷ 1000

so 2.5 km = 2.5 × 1000 = 2500 m

Taking care with decimal places is vital.

50 m = 0.05 km (not 0.5 as is commonly written when not being careful)

Students need to be able to estimate roughly the measurements of various things and what to measure them with. Some useful pieces of information:

A litre of water weighs 1 kg.

A cubic metre of water weighs 1 tonne.

MAN vs MATHS 2014

Time

The official metric unit of time is the second. Smaller divisions include milliseconds etc. However most questions will use common units, such as minutes and hours which are not decimal.

To convert time between seconds, minutes and hours requires multiplication or division by 60. Do **not** assume that you can convert as if it is a decimal conversion.

0.5 hours = 0.5 × 60 = 30 minutes (**not** 50 minutes)

150 seconds = 150 ÷ 60 = 2.5 minutes = 2 minutes 30 seconds

To avoid the confusing decimal portion it is preferable to do the division conversions by way of fractions.

230 seconds = $\frac{230}{60}$ = $3\frac{5}{6}$ min (with the a b/c button)

Any question requiring calculations using 230 seconds in minutes would use $3\frac{5}{6}$ min.

Rate

Rate is the mathematical word for the speed at which some amount is changing.

$$\text{rate} = \frac{\text{amount changed}}{\text{time taken}}$$

You can recognise a rate question by the units. A rate is given in terms of amount per time e.g. metres per second, often written as ms^{-1}, or pages per minute, which might be written p/min. As soon as the question is something related to time it will be a rate question.

Rate questions are always answered by multiplication and division. It generally helps to figure out how to do an easy one first with round numbers, and then use that same method with your numbers

Calculate how long it takes to mow a 265 m^2 lawn at 140 m^2 per hour:

It takes two hours to mow 200 m^2 at 100 m^2 per hour, so I must be dividing m^2 by rate.

So it takes 265 ÷ 140 = 1.89 hours

Students need to be particularly comfortable with calculating average speed, which is the rate of change of distance with time. Speed = $\frac{\text{distance}}{\text{time}}$

If a car travels 160 km in 1½ hours, then its average speed = 160 ÷ 1.5 = 107 km/hr

As with all calculations, it is important to convert every unit in the question before starting. That means converting all time to the same unit as well.

What is the rate per minute, if I cut 580 m^2 in 2 ½ hours?

2 ½ hours is 150 minutes, so the rate is 580 ÷ 150 = 3.87 m^2/min

MAN VS MATHS 2014

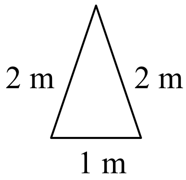
Perimeter

The perimeter of an object is the total distance traced around the edge of that object.

The perimeter of a circle is called the circumference, and is equal to $\pi \times$ diameter.

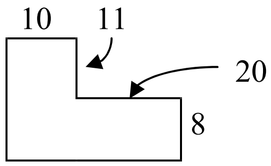
In some harder cases Pythagoras' Theorem will be needed to calculate the length of angled lines.

In simple cases the perimeter is given by adding all the distances given.



Perimeter = 2 + 2 + 1 = 5 m

In slightly harder cases some of the distances need to be calculated from those given.

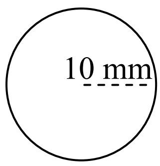


The missing vertical side must be 19 = 11 + 8

The missing horizontal side must by 30 = 10 + 20

Perimeter = 10 + 11 + 20 + 8 + 19 + 30 = 98 units

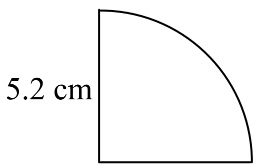
When calculating circumferences care must be taken to use the diameter. If the radius is given, then it should be doubled first.



Radius = 10 mm, so the diameter = $2 \times 10 = 20$ mm

Circumference = $\pi \times d = \pi \times 20 = 62.8$ mm

When calculating perimeters with an arc (portion of a circle) the circumference of the full circle is calculated, then the appropriate fraction applied. Do not forget the straight line components too.



The height given is the radius, so diameter = $5.2 \times 2 = 10.4$ cm

Curved line is quarter of full circle = $\frac{1}{4} \times \pi \times 10.4 = 8.17$ cm

Perimeter = curve + line + line = 8.17 + 5.2 + 5.2 = 18.57 cm

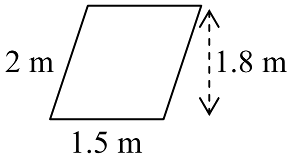
MAN vs MATHS 2014

Formulas for Area

The area of a rectangle or parallelogram = base × height.

The base can be any side. Height is always at 90° to the base.

Sometimes measurements of the object will not be required to calculate the area. (In particular the angled lines of triangles and parallelograms must be ignored and the 90° height used instead.)



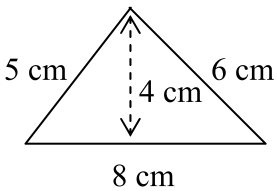
Area = base × height = 1.5 × 1.8 = 2.7 m^2

(the 2 m side length is used for perimeter, but is not a height)

The area of a triangle = ½ b h, where the base is any side and height is at 90° to that base.

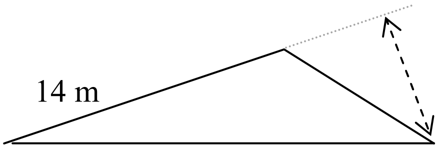
It is important to not forget the half for the triangle. This is a very common mistake. Some students prefer to halve at the end of the triangle formula: area = base × height ÷ 2

The height may not be one of the sides.



Area = ½ × base × height = ½ × 8 × 4 = 16 cm^2

(neither the 5 nor 6 are a height for the base of 8)

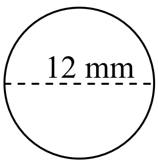


The dotted line is the height for the 14 m base.

The area of a circle = $\pi \times r^2$, where the r is the radius.

It is important not to confuse radius^2 with radius × 2. To prevent this confusion, some students prefer to remember the formula as: area = π × radius × radius.

If the diameter is given it must be halved to find the radius **first**, before anything else is calculated.



Diameter = 12 mm, so the radius = 12 ÷ 2 = 6 mm

Area = π × radius^2 = $\pi \times 6^2$ = 113.1 mm^2

MAN vs MATHS 2014

Calculations of Area

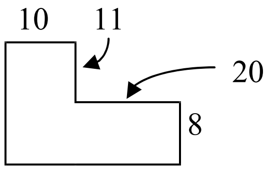
Area must be calculated with the same units for all measurements.

Any change of units must be done before anything else in the calculation. Most students find it difficult to grasp that 1 m^2 = 10,000 cm^2 not 100 cm^2.

The units of area are always squared, e.g. cm^2, m^2, except hectares. (1 ha = 100 m × 100 m.)

Complicated shapes can be calculated by breaking them down into smaller ones.

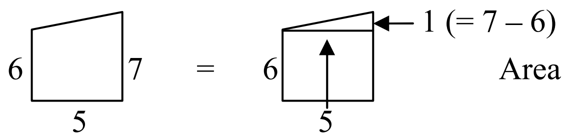
If all the lines are at 90°, then the object will be made up of a number of rectangles.



This can be broken into three smaller rectangles

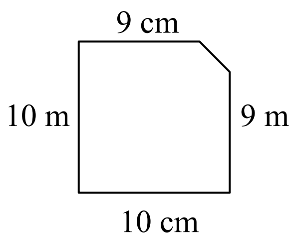
Area = (10 × 11) + (20 × 8) + (10 × 8) = 350 units^2

A trapezium can be broken into a rectangle and a triangle.



Area = (6 × 5) + (½ × 5 × 1) = 32.5 units^2

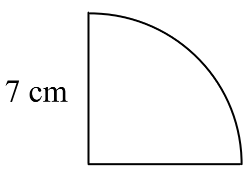
As well as calculating by adding smaller areas, a section removed can be done as a subtraction.



Area = square – triangle missing

= (10 × 10) – (½ × 1 × 1) = 99.5 cm^2

When calculating areas with an sector (slice of a circle) the area of the full circle is calculated, then the appropriate fraction applied.



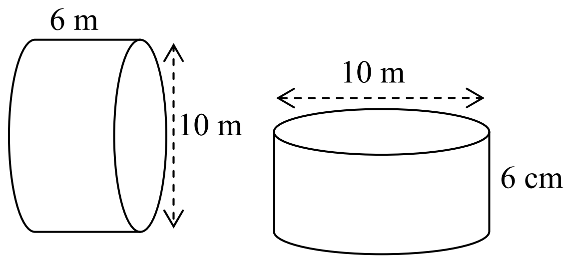
Area = ¼ of a circle = ¼ × π × 7^2 = 38.5 cm^2

MAN vs MATHS 2014

Volume

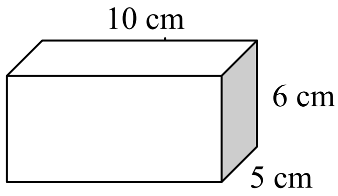
The volume of a prism is calculated by the base area × depth, where depth is the length at 90° to the regular base.

The “base area” is the regular face, whether on the bottom or not, and the “depth” can be a height.



Volume = base area × depth
$= \pi \times r^2 \times$ depth
$= \pi \times 5^2 \times 6 = 471.2$ cm^3
for both cylinders

If all the edges are at 90°, the object is a cuboid, and the volume = base × height × depth.

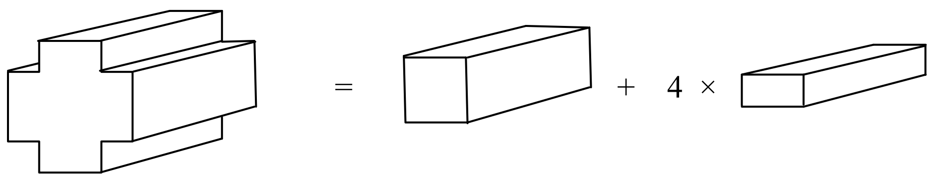


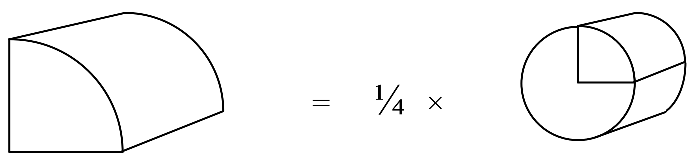
Volume = base area × depth = (5 × 6) × 10 = 300 cm^3

As with all measurement calculations, volume must be calculated with the same units for all measurements and any change of units must be done before anything else in the calculation.

The units of volume are always cubed, e.g. cm^3, m^3, except Litres. (1 mL = 1 cm^3)

As with area, complicated shapes can be calculated by breaking them down into smaller ones and portions of simple shapes are calculated as fractions of the whole.





The pointed versions of prisms (cones, pyramids) have a volume one third that of their matching prism versions, e.g. a cone = ⅓ $\pi r^2 h$. (Such shapes are not expected, except at higher levels.)

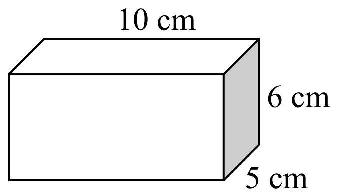
MAN vs MATHS 2014

Surface Area

Surface area is the sum of all the faces, including those hidden from view.

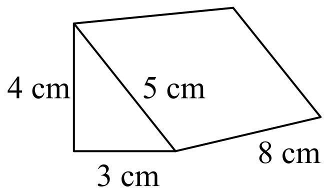
It often helps to draw a "net" of the object when calculating surface area.

A cuboid has six faces, all rectangles, in equal pairs: front = back, top = bottom and end = end.



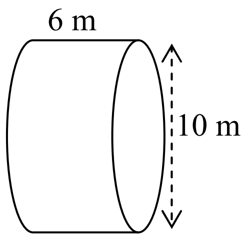
$$\text{Surface area} = [\,(6 \times 10) + (5 \times 10) + (5 \times 6)\,] \times 2 = 280 \text{ cm}^2$$

A triangular prism has five faces: the three rectangular sides and two triangular ends. It is important to remember to halve the areas of the triangles.



$$\text{Surface area} = (5 \times 8) + (4 \times 8) + (3 \times 8) + 2 \times (\tfrac{1}{2} \times 3 \times 4) = 108 \text{ cm}^2$$

A cylinder has two round ends and a rectangular face, which is the circle's circumference by the cylinder's height.



$$\text{Surface area} = 2 \text{ circles} + \text{side} = 2 \times (\pi \times r^2) + (\pi \times d \times h)$$
$$= 2 \times \pi \times 5^2 + \pi \times 10 \times 6 = 345.6 \text{ cm}^2$$

Surface area must be calculated with the same units for all measurements.

Any change of units must be done before anything else in the calculation.

It is still area, even if in three directions, so the units of surface area are squared, e.g. cm^2, m^2.

MAN vs MATHS 2014

Extension Material (never required for Achieved)

Limits of Accuracy

No measurement is entirely accurate. There is always some "observational error" associated with any measurement – errors by the person measuring, in the instrument being used, or associated with poor assumptions, such as standard temperature and gravity.

If the amount of error, unless stated, can be assumed to be ± half the last significant figure.

When calculating the error of area, volume etc, it is important to put the errors in at the **start** of the calculation.

The result is that any measurement is, in fact, a range.

A stated length of "64 m" is actually somewhere between 63.5 and 64.5 metres (because the last significant figure is whole centimetres, the error is ± 0.5 cm).

However a length of "64.0 m" is actually somewhere between 63.95 and 64.05 m (the addition of ".0" says it is to the nearest mm, so the error is ± 0.05 cm).

Errors build very quickly if the size of the error is large relative to the measurement, and doubly so if two large errors are multiplied.

A circle of "radius = 6 cm" has an actual radius between 5.5 and 6.5 cm.

The area can be as large as $\pi \times 6.5^2 = 132.7$ cm^2 and as small as $\pi \times 5.5^2 = 95.0$ cm^2

Which gives basically an area of 113 ± 20 cm^2 – a potential error of nearly 20%!

If taking difference measurements, it is important to take the low error from the high error and *vice versa*.

Temperature is measured at 26° at the hottest time of the day, and 8° at the coldest.

The temperature difference is 26 – 8 = 18°. However both readings have errors, and it can actually be as large as 26.5 – 7.5 = 19° and as small as 25.5 – 8.5 = 17°.

Therefore the correct difference is 18° ± 1°.

Generally you should do a calculation with the exact values first, to establish the method. Then afterwards do it with errors.

Often the error only needs to be considered in one direction e.g. if a question asks if something can be done in a time period, the errors that make it quicker can be ignored, and only the errors that make it slower need to be calculated.

Be alert for any indications that errors need to be considered, even if it is only by some indirect indication such as the problem saying "to the nearest 10 cm" or "to within 5 cm" with a measurement.

MAN vs MATHS 2014