L1 Algebra Trial #3

Q1. a) Show that for every value of x that $(x - 3)(x - 7)$ and $(x - 5)^2$ differ by exactly 4.

b) What number to the power of 5 is equal to 100,000?

c) The product of which two consecutive integers is the same as one and a half times the next consecutive number?

d) Bill works for 20 hours at a set pay rate. Sally works for 24 hours at a rate $2 less per hour. Sally makes less money than Bill. What can we say about Bill's rate?

e) What solution(s) are there for: $\frac{2}{x} + x = 3$?

f) Three equal sized rectangular fields sharing borders as shown are made with 120m of fencing. If their total area is 400 m², what are the dimensions of the fields?

Q2. a) Show that the graph of $y = 5x^2 - 36x + 7$ has x-intercepts at $x = 7$ and $x = \frac{1}{5}$

b) Find $P = \frac{2a + b}{a + 2b}$ if $a = 5$ and $b = {}^-2$.

c) Show that $x^2 - 8x + 16$ gives a square number for any integer x.

d) $x^2 + ax + 10 = (x + b)(x + c)$ where b and c are integers. What values can a have?

e) An adult ticket and a child ticket cost $22.50 and two adult tickets and three child tickets cost $52.50. How much is a child ticket?

f) Write a rule for the linear pattern whose 100th, 101st and 102nd terms are
... 7, 11, 15, ...

Q3. a) If $\frac{3}{4x^3y} = \frac{a}{8x}$ what is a equal to in terms of x and y ?

b) Write a quadratic expression that gives values greater than zero only when x is less than 2 or more than 5.

c) Rewrite $\frac{3}{x} + \frac{1}{2x}$ as a single (fractional) term.

d) Give k in terms of y for $y = (k - 2)^2$

e) Steve is two years older than Bill. If their ages multiplied is 440, how old is Steve?

f) Show that the difference between any two odd numbers is an even number.
(*Hint: any odd number can be written as* 2n + 1, *where* n *is an integer.*)

L1 Algebra Trial #3 : Answers

Colours indicate the **approximate** point when Achieved, Merit and Excellence are reached.

Q1. a) $(x - 3)(x - 7) = x^2 - 10x + 21$ and $(x - 5)^2 = (x - 5)(x - 5) = x^2 - 10x + 25$
which are 4 different

b) Solve $x^5 = 100,000$ $x = 10$

c) Solve $x(x + 1) = 1.5(x + 2)$ Doubling both sides $2x(x + 1) = 3(x + 2)$
$2x^2 + 2x = 3x + 6$ $2x^2 - x - 6 = 0$ $(2x + 3)(x - 2) = 0$
$x = 2$ or a non-integer $(-2/3)$ The numbers are 2 and 3

d) Solve $20x > 24(x - 2)$ $20x > 24x - 48$ $48 > 24x - 20x$
$48/4 > x$ $12 > x$ Bill earns less than \$12 per hour

e) $\frac{2}{x} + x = 3$ multiply through by x gives $2 + x^2 = 3x$ (because $\frac{2x}{x} = 2$)
$x^2 - 3x + 2 = 0$ $(x - 1)(x - 2) = 0$ $x = 1$ or 2

f) Area = b × h. Let x be the h, then b = ½(120 − 4x) = 60 − 2x
Area = 400 = x(60 − 2x) 400 = 60x − 2x² (÷ 2) $x^2 - 30x + 200 = 0$
$(x - 10)(x - 20) = 0$ fields are 10m × 40m or 20m × 20m

Q2. a) $5x^2 - 36x + 7 = (5x - 1)(x - 7)$ so $y = 0$ if $5x - 1 = 0$ or $x - 7 = 0$
So when $x = 7$ and $x = \frac{1}{5}$

b) $P = \frac{10 + -2}{5 + -4} = \frac{8}{1}$ $P = 8$

c) $x^2 - 8x + 16 = (x - 4)^2$ since $x - 4$ is an integer if x is an integer, we always
get an integer squared for every integer x, so a square number.

d) $b\, c = 10$, so the possible pairs of values are 1 × 10, 2 × 5, ⁻2 × ⁻5, ⁻1 × ⁻10
As $a = b + c$ we see $a = $ 7, 11, ⁻7 or ⁻11 (A if only 7 and 11)

e) a + c = 22.50 so a = 22.50 − c and 2a + 3c = 52.50 (need to use equations)
So 2(22.50 − c) + 3c = 52.50 45 − 2c + 3c =52.50 child = \$7.50

f) ①100 × k + c = 7 and ②101 × k + c = 11, so ② − ① gives $k = 4$ (the multiplier)
Solve 100 × 4 + c = 7, $c = -393$ (the constant) Rule is $t_n = 4n - 393$

Q3. a) $\frac{3}{4x^3y} = \frac{a}{8x}$ so $a = \frac{3 \times 8x}{4x^3y} = \frac{4x \times 6}{4x \times x^2y}$ $a = \frac{6}{x^2y}$

b) $(x - 2)(x - 5)$ or $x^2 - 7x + 10$ as a parabola with intercepts at $x = 2$ and 5

c) $\frac{3}{x} + \frac{1}{2x}$ $= \frac{6}{2x} + \frac{1}{2x}$ $= \frac{7}{2x}$

d) $y = (k - 2)^2$ $\pm\sqrt{y} = k + 2$ $k = 2 \pm \sqrt{y}$ or $k = \pm\sqrt{y} + 2$ (A if no ±)

e) B × S = 440 so (S − 2)S = 440 S² − 2S − 440 = 0 (S − 22)(S + 20) = 0
S = 22 or −20. Negative age makes no sense Steve is 22

f) Difference of two odd numbers = (2n + 1) − (2m + 1) where n and m integers
difference is 2n + 1 − 2m − 1 = 2n − 2m = 2(n − m)
As n and m are integers, n − m is an integer, 2 × any integer must be even