L1 Algebra Trial #3

Q1. a) Solve: $10x + 13 = 3x - 8$

b) Solve: $x^5 = 100{,}000$

c) Solve: $3x + 7 \geq 7x - 11$

d) Solve: $x(x + 2) = 5(x + 2)$

e) Solve: $\frac{2}{x} + x = 3$

f) 

Three equal sized rectangular fields are made with 120m of fencing. If their total area is 400 m^2, what are the dimensions of the fields?

Q2. a) Factorise: $x^2 - 8x + 16$

b) Find $P = \frac{2a + b}{a + 2b}$ = if $a = 5$ and $b = {}^{-}2$:

c) Simplify fully: $\frac{2x^2 + 10x}{4x}$

d) $x^2 + ax + 10 = (x + b)(x + c)$ where b and c are integers. What values can a have?

e) An adult ticket and a child ticket cost $22.50 and two adult tickets and three child tickets cost $52.50. How much is a child ticket?

f) Write a rule for the linear pattern whose 100^{th}, 101^{st} and 102^{nd} terms are … 7, 11, 15, …

Q3. a) Simplify: $10x \,.\, 2y^2 \div 5x^2y$

b) Expand: $(3 - x)(4 - x)$

c) Simplify to one fraction: $\frac{3}{x} + \frac{1}{2x}$

d) Make k the subject: $y = (k - 2)^2$

e) Steve is two years older than Bill. If their ages multiplied is 440, how old is Steve?

f) Show that the difference between any two odd numbers is an even number.
(*Hint: any odd number can be written as* $2n + 1$, *where* n *is an integer.*)

L1 Algebra Trial #3 : Answers

In general terms: a) & b) are Achieved, c) & d) are Merit, e) & f) are Excellence

Q1. a) Solve: $10x + 13 = 3x - 8$ $\quad 10x - 3x = {}^-8 - 13 \quad 7x = {}^-21 \quad$ $\boldsymbol{x = {}^-3}$

b) Solve: $x^5 = 100{,}000$ $\quad$ $\boldsymbol{x = 10}$

c) Solve: $3x + 7 \geq 7x - 11$ $\quad 7 + 11 \geq 7x - 3x \quad 18 \div 6 \geq x \quad$ $\boldsymbol{x \leq 4.5}$

d) Solve: $x(x + 2) = 5(x + 2)$ $\quad x^2 + 2x = 5x + 10 \quad x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$ $\quad$ $\boldsymbol{x = {}^-2}$ **or 5** (need both)

e) Solve: $\dfrac{2}{x} + x = 3$ $\quad$ multiply through by x gives $2 + x^2 = 3x$ (because $\dfrac{2x}{x} = 2$)

$x^2 - 3x + 2 = 0 \qquad (x - 1)(x - 2) = 0 \qquad$ $\boldsymbol{x = 1}$ **or 2**

f) 

Three equal sized rectangular fields are made with 120m of fencing. If their total area is 400 m^2, how wide are the fields?

Area = b × h. Let x be the h, then b =½(120 – 4x) = 60 – 2x

Area = 400 = $x(60 - 2x)$ $\quad 400 = 60x - 2x^2 \quad (\div 2)\ x^2 - 30x + 200 = 0$

$(x - 10)(x - 20) = 0.$ $\quad$ **fields are 10m × 40m or 20m × 20m**

Q2. a) Factorise: $x^2 - 8x + 16$ $\quad$ $\boldsymbol{= (x - 4)(x - 4)}$ **or** $\boldsymbol{(x - 4)^2}$

b) Find $P = \dfrac{2a + b}{a + 2b}$ = if $a = 5$ and $b = {}^-2$: $\quad P = \dfrac{10 + -2}{5 + -4} = \dfrac{8}{1}$ $\quad$ $\boldsymbol{P = 8}$

c) Simplify fully: $\dfrac{2x^2 + 10x}{4x}$ $\quad = \dfrac{2x\,(x + 5)}{2x \times 2}$ $\quad = \boldsymbol{\dfrac{x + 5}{2}}$

d) $x^2 + a\,x + 10 = (x + b)(x + c)$ where b, and c are integers. What values can a be?

$b\ c = 10$, so the possible pairs of values are 1×10, 2×5, ${}^-2 \times {}^-5$, ${}^-1 \times {}^-10$

As $a = b + c$ we see $\boldsymbol{a = 7, 11, {}^-7}$ **or** $\boldsymbol{{}^-11}$

e) An adult ticket and a child ticket cost \$22.50 and two adult tickets and three child tickets cost \$52.50. How much is a child ticket?

a + c = 22.50 so a = 22.50 – c and 2a + 3c = 52.50 (need to use equations)

So 2(22.50 – c) + 3c = 52.50 45 – 2c + 3c =52.50 **child = \$7.50**

f) Write a rule for the pattern whose 100th, 101st and 102nd terms are … 7, 11, 15, …

①100 × $k + c = 7$ and ②101 × k + c = 11, so ② – ① gives $k = 4$ (the multiplier)

Solve 100 × 4 + c = 7, c = −393 (the constant) $\quad$ **Rule is $\boldsymbol{t_n = 4n - 393}$**

Q3. a) Simplify: $10x \,.\, 2y^2 \div 5x^2y$ $= \dfrac{4y \times 5xy}{1x \times 5xy} = \dfrac{4y \times \cancel{5xy}}{x \times \cancel{5xy}}$ $= \boldsymbol{\dfrac{4y}{x}}$ or $4yx^{-1}$

b) Expand: $(3 - x)(4 - x)$ $\quad = 12 - 3x - 4x + x^2$ $\quad$ $\boldsymbol{= x^2 - 7x + 12}$ (any order)

c) Simplify to one fraction: $\dfrac{3}{x} + \dfrac{1}{2x}$ $\quad = \dfrac{6}{2x} + \dfrac{1}{2x}$ $\quad = \boldsymbol{\dfrac{7}{2x}}$

d) Make k the subject: $y = (k - 2)^2$ $\quad \pm\sqrt{y} = k + 2$ $\quad$ $\boldsymbol{k = 2 \pm\sqrt{y}}$ or $k = \pm\sqrt{y} + 2$

e) Steve is two years older than Bill. If their ages multiplied is 440, how old is Steve?

B × S = 440 so (S – 2)S = 440 $\quad$ S^2 – 2S – 440 = 0 $\quad$ (S – 22)(S + 20) = 0

S = 22 or −20. Negative age makes no sense $\quad$ **Steve is 22**

f) Show that the difference between any two odd numbers is an even number.
(*Hint: any odd number can be written as* 2n + 1, *where* n *is an integer.*)

Difference of two odd numbers = (2n + 1) – 2(m + 1) where n and m integers

difference is 2n + 1 – 2m – 1 = 2n – 2m = 2(n – m)

As n and m are integers, n – m is an integer, **2 × any integer must be even**