L1 Algebra Trial #5

Q1. a) Simplify: $(5x^2y)^2$

   b) Expand: $2(x + 2)(x - 3)$

   c) Simplify to one fraction: $\dfrac{3}{x} + y$

   d) Simplify: $\sqrt{64/x^6}$

   e) Solve: $\dfrac{7}{x + 2} + 4 = x$

   f)   The number of intersections has the pattern 0, 1, 3, 6, 10, 15 … Give an equation for the intersections in terms of the number of lines.

Q2. a) Solve: $\sqrt{x - 1} = 8$

   b) Find $k = a^2 - b$ if $a = {}^-2$ and $b = {}^-6$:

   c) Solve: $3 < 5x - 12$

   d) Solve: $x^4 + 10 = 26$

   e) If three red buttons weigh as much as 5 blue buttons, and four red buttons weigh 140 grams more than 2 blue buttons, what does a red button weigh?

   f) A rectangle is 5 cm longer than it is high. A 1 cm border is added all the way around. The border has an area of 26 cm². What is the size of the rectangle inside?

Q3. a) Factorise: $x^2 + 8x - 20$

   b) Solve: $5^x = 125$

   c) Simplify fully: $\dfrac{x^2 + 10x + 24}{2x + 8}$

   d) Solve: $0.5x^2 = 4x - 7.5$

   e) Solve: $x^4 = 25x^2$

   f) Show that an odd number times the next consecutive odd number is one less than the even number between them squared.

L1 Algebra Trial #5 : Answers

Q1. a) Simplify: $(5x^2y)^2$ $= 5x^2y \times 5x^2y$ $\mathbf{25\,x^4 y^2}$

b) Expand: $2(x+2)(x-3)$ $= 2(x^2 + 2x - 3x - 6)$ $\mathbf{= 2x^2 - 2x - 12}$

c) Simplify to one fraction: $\frac{3}{x} + y$ $= \frac{3}{x} + \frac{xy}{x}$ $= \mathbf{\frac{3 + xy}{x}}$

d) Simplify: $\sqrt{64/x^6}$ $= \sqrt{64}/\sqrt{x^6}$ $= \mathbf{\frac{8}{x^3}}$ or $8\,x^{-3}$

e) Solve: $\frac{7}{x+2} + 4 = x$ $\frac{7}{x+2} = x - 4$ $7 = (x-4)(x+2)$

$x^2 - 2x - 15 = 0$ $(x-5)(x+3) = 0$ $\mathbf{x = {}^-3 \text{ or } 5}$

f)   The number of intersections has the pattern 0, 1, 3, 6, 10, 15 … Give an equation for the intersections in terms of the number of lines.

Pattern increases by +1, +2, +3 so it is a quadratic, at half n^2 rate of +1

i = ½n^2 is 0.5, 2, 4.5, 8, … so too high by 0.5, 1, 1.5, 2 … which is ½n.

i = ½n^2 − ½n or i = ½ n(n − 1)

Q2. a) Solve: $\sqrt{x-1} = 8$ $x - 1 = 64$ $\mathbf{x = 65}$

b) Find $k = a^2 - b$ if $a = {}^-2$ and $b = {}^-6$: $k = ({}^-2)^2 - {}^-6$ $= 4 + 6$ $\mathbf{k = 10}$

c)   Solve: $3 < 5x - 12$ $3 + 12 < 5x$ $15 \div 5 < x$ $\mathbf{x > 3}$

d) Solve: $x^4 + 10 = 26$ $x^4 = 16$ $\mathbf{x = \pm 2}$ (need ± for M)

e) If three red buttons weigh as much as 5 blue buttons, and four red buttons weigh 140 grams more than 2 blue buttons, what does a red button weigh?

3r = 5b so b = $^3/_5$ r   and   4r = 2b + 140   (need to use equations)

So 4r = 2(0.6 r) + 140   2.8 r = 140   **r = 50**

f) A rectangle is 5 cm longer than it is high. A 1 cm border is added all the way around. The border has an area of 26 cm^2. What is the size of the rectangle inside?

If the rectangle is x high, Area = $x(x+5)$. Area with border = $(x+2)(x+7)$

26 = $(x+2)(x+7) - x(x+5)$ 12 = $x^2 + 9x + 14 - x^2 - 5x$

26 = $4x + 14$, so $x = 3$.   **The rectangle is 3 cm by 8 cm**

Q3. a) Factorise: $x^2 + 8x - 20$ $\mathbf{= (x-2)(x+10)}$

b) Solve: $5^x = 125$ $\mathbf{x = 3}$

c) Simplify fully: $\frac{x^2 + 10x + 24}{2x + 8}$ $= \frac{(x+6)(x+4)}{2(x+4)}$ $= \mathbf{\frac{x+6}{2}}$

d) Solve: $0.5x^2 = 4x - 7.5$ $x^2 - 8x + 15 = 0$ $(x-3)(x-5) = 0$ $\mathbf{x = 3 \text{ or } 5}$

e) Solve: $x^4 = 25x^2$ $x^4 - 25x^2 = 0$ $x^2(x^2 - 25) = 0$ $x^2 = 0$ or $x^2 - 25 = 0$

$x = 0$ or $(x-5)(x+5) = 0$ $\mathbf{x = 0, 5 \text{ or } {}^-5}$

f) Show that an odd number times the next consecutive odd number is one less than the even number between them squared.

Let the first number be x. So that times the next odd number is $x(x+2) = x^2 + 2x$

The even number between them is $x + 1$, so squared is $(x+1)^2 = x^2 + 2x + 1$

We see that $(x + 1)^2 = x^2 + 2x + 1$ is one more than $x(x + 2) = x^2 + 2x$