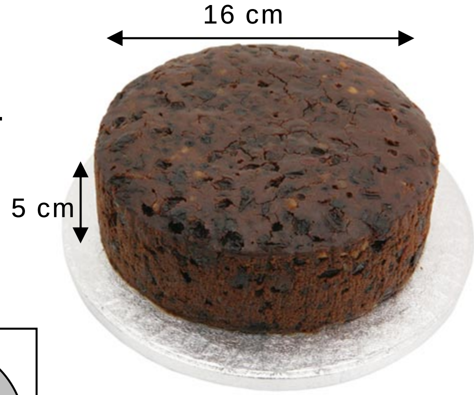
Trial Measurement #1 – "Cakes"

Useful formulas:

Area of a circle = πr^2 — Circumference of a circle = πd

Area of a parallelogram = b x h

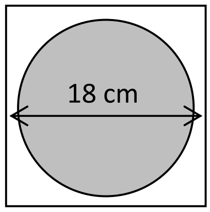
Area of a triangle = ½ b h — $\pi = 3.14159$

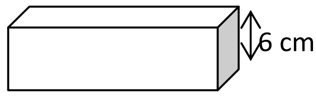


Sally sells cakes. They are round, 16 cm across, and 5 cm high.

Sally puts them in square boxes, which are 18 cm square.
(She wants to leave at least a 1 cm gap on either side, so they don't stick to the edge.)

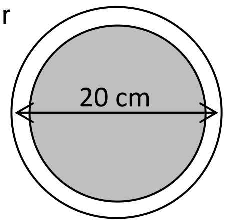
The boxes are 6 cm high, which leaves some room so the tops don't stick.

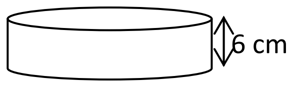




She is thinking of trying some different packing systems. One is to use cylinder boxes.

The only suitable boxes for sale are 20 cm in diameter and 6 cm high.



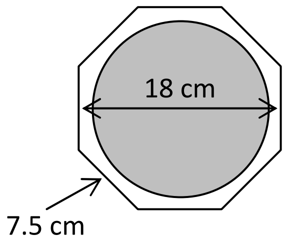


Another possibility is to use an octagonal prism shape.

It is still has to be at least 18 cm across from side to side so that she still has the room spare around the outside.

She discovers that each side of the octagon needs to be 7.5 cm to do this.

The box is still 6 cm high.



Task One

Which of the boxes leaves the least amount of air in each box after a cake is put in?

Task Two

Which of the boxes uses the least amount of cardboard to build?

Task Three

Find how many boxes (of whichever sort you prefer) Sally can pack in her storage area.

The area is 80 cm wide and 1.2 metres deep. She can only stack the cakes two deep, or they crush the ones underneath.

MAN VS MATHS 2013

Answers: Trial Measurement #1

Each cake is $\pi \times 8^2 \times 5 = 1005 \text{ cm}^3$ in volume | A

The “square” box has a volume of $18 \times 18 \times 6 = 1{,}944 \text{ cm}^3$

So that box has $1944 – 1005 = 939 \text{ cm}^3$ of air. A

The “round” box has a volume of $\pi \times 10^2 \times 6 = 1{,}885 \text{ cm}^3$

So that box has $1885 – 1005 = 880 \text{ cm}^3$ of air. A

The octagon shape has an area of 270 cm^2 (see bottom of page) so $270 \times 6 = 1620 \text{ cm}^3$

So the octagon box has the least air, at $1620 – 1005 = 615 \text{ cm}^3$ M

The surface area of the square box is

$2 \times (18 \times 18) + 2 \times (6 \times 18) + 2 \times (6 \times 18) = 1080 \text{ cm}^2$ A

The surface area of the round box is $2 \times (\pi \times 10^2) + \pi \times 20 \times 6 = 1005 \text{ cm}^2$ A

The surface area of the octagonal box is 270 (below) + $270 + 8 \times 6 \times 7.5 = 900 \text{ cm}^2$ A

So the octagonal box uses the least cardboard. M

Sally can fit

$80 \div 18 = 4.44$, so four boxes across

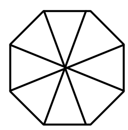
$120 \div 18 = 6.666$, so six boxes deep

Two boxes high.

So with the square boxes she can fit $4 \times 6 \times 2 = 48$ boxes A

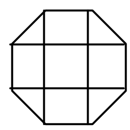
She can fit exactly the same number of round and octagonal ones. M

The octagon is best worked out at 8 equal triangles from the centre.



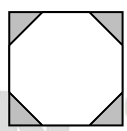
$8 \times (½ \times b \times h) = 8 \times ½ \times 7.5 \times 9 = 270 \text{ cm}^2$

You can make a square in the centre, four rectangles and four triangles around it:



$7.5 \times 7.5 + 4 \times (5.25 \times 7.5) + 4 \times (½ \times 5.25 \times 5.25) = 269 \text{ cm}^2$

You can take the outside square and deduct the four triangles from the corners:



$18 \times 18 – 4 \times (½ \times 5.25 \times 5.25) = 269 \text{ cm}^2$

(The difference is because the sides aren’t exactly 7.5 long, but actually 7.456 cm)