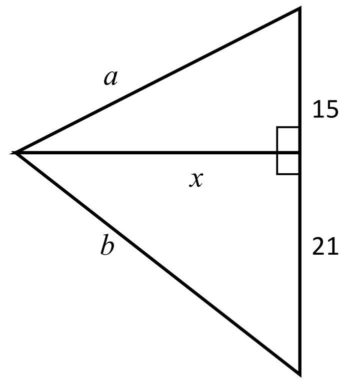
Year 12 Algebra Excellence #1

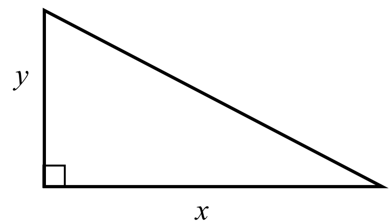
1. Find x so that b is 4 more than a.



2. Factorise: $3^{x+3} - 9^x$

3. Write an expression for x in terms of y

   if the sum of x and y is equal to the triangle's area.



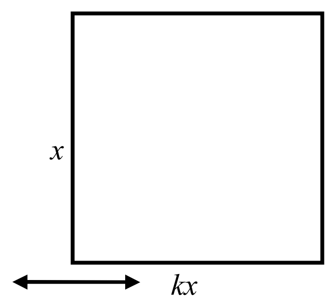
4. Solve: $x + \sqrt{x} = 156$

5. A square has side length x.

   It is changed to a rectangle by multiplying one of the sides by a multiple, k.

   Find k in terms of x, if the area of the new rectangle is the same number as the perimeter.

   Simplify your answer fully.



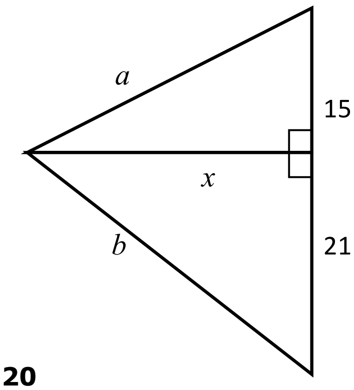
6. Find k so that $y = 4x^2 + k\,x + 15$ has two x-intercepts that are exactly 1 apart.

7. Make x the subject of the equation: $4^{x+1} = k^x$

8. Find both the solutions to: $a^2 + ab = 2b^2$ for a in terms of b

Answers: Year 12 Algebra Excellence #1

1. Find x so that b is 4 more than a.



$x^2 + 15^2 = a^2$ and $x^2 + 21^2 = (a + 4)^2$

$x^2 = a^2 - 15^2$ and $x^2 = (a + 4)^2 - 21^2$

$a^2 - 225 = a^2 + 8a + 16 - 441$

$a = 25$

$x^2 + 15^2 = a^2$ so $x^2 + 225 = 625$ **x = 20**

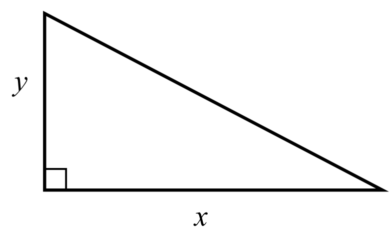
2. Factorise $3^{x+3} - 9^x$

$= 3^{x+3} - 3^{2x}$

$= 3^x(3^3 - 3^x)$

$= \mathbf{3^x(27 - 3^x)}$

3. Write an expression for x in terms of y if the sum of x and y is equal to the triangle's area.



$x + y = ½\,x\,y$

$x - ½\,x\,y = -y$

$x\,(1 - ½\,y) = -y$

$x = \dfrac{-y}{1 - \frac{1}{2}y}$

$\boldsymbol{x = \dfrac{-2y}{2 - y}}$ or $x = \dfrac{2y}{y - 2}$

4. Solve $x + \sqrt{x} = 156$

$\Rightarrow \quad x + \sqrt{x} - 156 = 0$

$\Rightarrow \quad (\sqrt{x} + 13)(\sqrt{x} - 12) = 0$

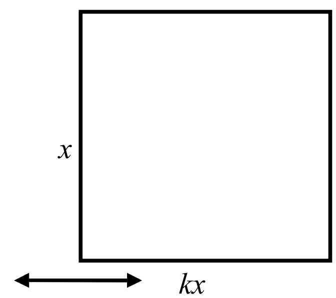
$\Rightarrow \quad \sqrt{x} = {}^-13$ or $\sqrt{x} = 12$

first is not possible, second when **x = 144**

5. A square has side length x.

It is changed to a rectangle by multiplying one of the sides by a multiple, k.

Find k in terms of x, if the area of the new rectangle is the same number as the perimeter.



$kx^2 = 2x + 2kx \quad \Rightarrow \quad k\,(x^2 - 2x) = 2x$

$k = \dfrac{2x}{x^2 - 2x} \quad \Rightarrow \quad \boldsymbol{k = \dfrac{2}{x - 2}}$

6. Find k so that $y = 4x^2 + k\,x + 15$ has two x-intercepts that are exactly 1 apart.

x-intercepts on graph are roots of the factorised equation.

$\Rightarrow \quad 4x^2 + k\,x + 15 = 4(x - \mathrm{r})(x - \mathrm{r} + 1)$ r is one root, so r + 1 is the other

$\Rightarrow \quad 4x^2 + k\,x + 15 = 4x^2 + (8\mathrm{r} + 4)x + (4\mathrm{r}^2 + 4\mathrm{r})$

coefficients must match $\Rightarrow 4\mathrm{r}^2 + 4\mathrm{r} = 15 \quad \Rightarrow \quad \mathrm{r} = 1.5$ or $^-2.5$

again coefficients must match $\Rightarrow k = 8\mathrm{r} + 4 \quad \Rightarrow \quad$ **k = $^-$16 or 16**

7. Make x the subject of the equation: $4^{\,x + 1} = k^{\,x}$

$\Rightarrow \quad (x + 1)\log 4 = x \log(k)$ or $\Rightarrow x + 1 = x \log_4(k)$

$\Rightarrow \quad x\log 4 - x\log k = {}^-\log 4$ $\Rightarrow x - x\log_4(k) = {}^-1$

$\Rightarrow \quad x(\log 4 - \log k) = {}^-\log 4$ $\Rightarrow \boldsymbol{x = \dfrac{-1}{1 - \log_4 k}} = \dfrac{1}{\log_4 k - 1}$

$\Rightarrow \quad \boldsymbol{x = \dfrac{-\log 4}{\log 4 - \log k}} = \dfrac{\log 4}{\log k - \log 4} = \dfrac{\log 4}{\log(\frac{k}{4})}$ etc

8. Find both the solutions to: $a^2 + ab = 2b^2$ for a in terms of b

$a^2 + ab = 2b^2 \quad \Rightarrow a^2 + ab + \tfrac{1}{4}b^2 = 2b^2 + \tfrac{1}{4}b^2$

This is "completing the square" because $(a + \tfrac{1}{2}b)^2 = a^2 + ab + \tfrac{1}{4}b^2$

$\Rightarrow (a + \tfrac{1}{2}b)^2 = 2.25b^2 \quad \Rightarrow \quad a + 0.5b = \pm\sqrt{2.25}\sqrt{b^2} = \pm 1.5b$ Note ±

$a = -1.5b - 0.5b$ or $+1.5b - 0.5b$ $\boldsymbol{a = -2b}$ **or** $\boldsymbol{b}$

or using the quadratic equation: $a = \dfrac{-b \pm \sqrt{b^2 - 4a(-2b^2)}}{2 \times 1} = \dfrac{-b \pm \sqrt{9b^2}}{2} = \dfrac{-b \pm 3b}{2} = \boldsymbol{-2b}$ **or** $\boldsymbol{b}$