Year 12 Algebra Excellence #6

1. Solve: $3^x = 6^{x-2}$

2. Simplify fully: $\frac{1}{a^3 - a} + \frac{a}{1 - a^2}$

3. Solve: $(x + 3)^2 > (2x + 1)^2$

4. Write an expression for difference between the solutions of $y = 5x^2 + 9x + k$

   Include any limits to your solution.

5. A bank pays interest at 6% p.a. Write an expression in terms of m, the number of months the money is invested, for a monthly rate that gives exactly result as paying 6% at the end of the year for a sum of $1000 invested (and untouched).

6. A ball is thrown in a parabola on level ground so that goes 30 metres distance, reaching a maximum height of 12 metres. For what proportion of the time is it above 9 metres?

7. Rewrite $k = 5^{2x-1}$ to make x the subject, and simplify fully to a single log term.

8. Show that $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} = \frac{1}{\log_{60} x}$

Answers: Year 12 Algebra Excellence #6

1. Solve $3^x = 6^{x-2}$

$\Rightarrow 3^2 \times 3^{x-2} = 2^{x-2} \times 3^{x-2}$

Note $6^{x-2} = 2^{x-2} \times 3^{x-2}$ as each 6 is 3 × 2

$\Rightarrow 3^2 \times 3^{x-2} = 2^{x-2} \times 3^{x-2}$

$\Rightarrow \log 9 = (x - 2) \log 2$

$\Rightarrow x = \log 9 \div \log 2 + 2$

Answer x = 5.17

or $\log(3^x) = \log(6^{x-2})$

$x \log 3 = (x - 2) \log 6$

$x \log 3 = x \log 6 - 2 \log 6$

$x (\log 3 - \log 6) = -2 \log 6$

$x = -2 \log 6 \div (\log 3 - \log 6)$

2. Simplify fully: $\frac{1}{a^3 - a} + \frac{a}{1 - a^2}$

$= \frac{1}{a^3 - a} + \frac{a^2}{a - a^3}$ $= \frac{1}{a^3 - a} - \frac{a^2}{a^3 - a}$ $= \frac{1 - a^2}{a - a^3}$ $= \frac{-1\,(a^2 - 1)}{a\,(a^2 - 1)}$

$\mathbf{= \frac{-1}{a}}$

3. Solve: $(x + 3)^2 > (2x + 1)^2$

$\Rightarrow \quad x^2 + 6x + 9 > 4x^2 + 4x + 1 \quad \Rightarrow \quad 0 > 3x^2 - 2x - 8$

$\Rightarrow \quad (3x + 4)(x - 2) < 0$ which happens when only one bracket is negative

Answer: $\mathbf{\frac{-4}{3} < x < 2}$

4. Write an expression for difference between the solutions of $y = 5x^2 + 9x + k$

General solutions to a quadratic are via the quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-9 \pm \sqrt{9^2 - 4 \times 5 \times k}}{2 \times 5}$

The difference between the solutions is + solution minus the − solution

$\Delta x = \frac{-9 + \sqrt{9^2 - 4 \times 5 \times k}}{2 \times 5} - \frac{-9 - \sqrt{9^2 - 4 \times 5 \times k}}{2 \times 5} = \frac{2\sqrt{81 - 20k}}{10}$

$\mathbf{\Delta x = 0.2\sqrt{81 - 20k}}$

But if $81 - 20k < 0$, then there are no solutions to have a difference of:

true only for $k \leq 4.05$

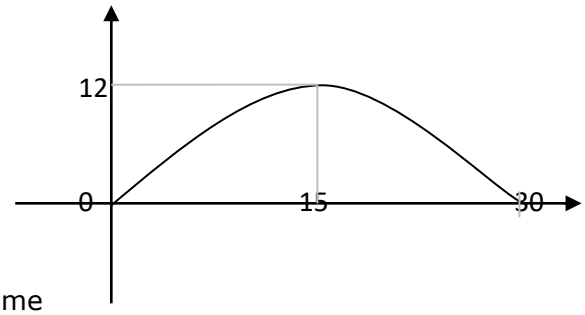
5. Calculate what annual interest would need to be paid on a bank balance if it was to exactly double in ten years.

monthly rate^{12} = 1.06 is the basic situation

r = $\sqrt[12]{1.06}$ = 1.004867

Equation is: **Sum = 1000 × 1.004867^{m}**

6. A ball is thrown in a parabola on level ground so that goes 30 metres distance, reaching a maximum height of 12 metres. For what percentage of the time is it above 9 metres?



h = $k\,x(x - 30)$

12 = $k \times 15(15 - 30)$ $\Rightarrow k = \frac{-4}{75}$

Need to solve 9 = $\frac{-4}{75}\,x(x - 30)$

x = 7.5 and 22.5

The ball is over 9 metres for $\frac{22.5 - 7.5}{30}$ of the time

Answer = 50% of the time

7. Rewrite $k = 5^{2x-1}$ to make x the subject, and simplify fully to a single log term

Using our formula sheet $y = b^{x}$ means $x = \log_b y$

$k = 5^{2x-1}$ $\Rightarrow$ $\log_5 k = 2x - 1$

$\Rightarrow$ $2x = \log_5 k + 1$

$\Rightarrow$ $2x = \log_5 k + \log_5 5 = \log_5 (5k)$

$\Rightarrow$ $x = ½ \log_5 (5k)$

$x = \log_5 (\sqrt{5k})$

8. Show that $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} = \frac{1}{\log_{60} x}$

Helps to know that $\log_b x = \frac{\log_{10} x}{\log_{10} b}$ so $\frac{1}{\log_b x} = \frac{\log_{10} b}{\log_{10} x}$

$\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x}$

$= \frac{\log_{10} 3}{\log_{10} x} + \frac{\log_{10} 4}{\log_{10} x} + \frac{\log_{10} 4}{\log_{10} x}$ $= \frac{\log_{10} 3 + \log_{10} 4 + \log_{10} 4}{\log_{10} x}$

$= \frac{\log_{10}(3\times4\times5)}{\log_{10} x}$ $= \frac{\log_{10} 60}{\log_{10} x}$ $= \frac{1}{\log_{60} x}$