Level 2 Algebra

When approaching any of these topics, the following might be kept in mind.

1) Work from what you know.

   Building from simple cases up helps when confusion might arise. Go back to a case you do know and then work up.

2) Almost all algebraic answers can be checked.

   "Solve" questions should give the right values when your answer is substituted back in.

   Rearrangement questions can be checked using a value for the unknown (10 is usually the best value to use, because it is easy to calculate powers quickly).

3) There are no marks for style, so any algebraic working that gives the correct answer is OK.

   Do not scribble out incorrect working. Put a box around it and one line though it. You may be given credit for incorrect answers if you working is clear and legible.

   You cannot get credit for scribbled working that only you can follow.

4) You may **not** answer questions in the Algebra paper with "guess and check" methods.

   Nor can you use the "solver" function on the calculator to solve equations. It can be useful for checking answers, however.

5) Learning skills by heart allows you to concentrate on the problem you face, instead of having to focus on trivial details of manipulation.

6) Merit and Excellence questions often require you to answer in context, and you will lose a lot if you do not do this. After finding the algebraic answer re-read the question to ensure you are answering what they asked you to answer.

Level 2 Algebraic Fractions

For Achieved you should to be able to do the following:

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1) Add algebraic fractions with different denominators:

A question will read: Simplify $\frac{2x}{y} + \frac{x}{y+1}$

and you have to give $\frac{3xy + 2x}{y^2 + y}$.

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2) Separate out common factors and cancel them out:

A question will read: Simplify $\frac{x^2 + 2x + 1}{x^2 - 1}$

and you have to give: $\frac{x+1}{x-1}$.

Merit questions may involve more steps at Achieved level, but there are no further techniques required.

"Simplify" does not mean the answer will be a simple fraction, but that the answer should be in the form of a single fraction with all common factors cancelled out.

Algebraic fractions obey exactly the same rules as normal fractions. It is hard to answer algebraic fraction questions if you are not comfortable with basic fractions.

Any expression on the top line or bottom line of a fraction being effectively in brackets for the purposes of BEDMAS.

Basics for Fractions

❶ Any number is converted to a fraction by putting it over 1.

$$5x \quad = \frac{5x}{1}$$

❷ When two fractions are multiplied, the answer is found by multiplying the two numerators and multiplying the two denominators.

$$\frac{a}{x} \times \frac{b}{y} \qquad = \frac{a \times b}{x \times y} \qquad = \frac{ab}{xy}$$

❸ The reverse is true, so you can separate out multiplication factors.

$$\frac{3x}{4} \qquad = \frac{3 \times x}{4 \times 1} \qquad = \frac{3}{4} \times \frac{x}{1} \qquad = \frac{3}{4}x$$

❹ The bottom line of a fraction can be changed by multiplying top and bottom by the same number. That number can also be an unknown.

$$\frac{x}{y} \quad = \frac{3 \times x}{3 \times y} \quad = \frac{3x}{3y} \qquad \text{and} \qquad \frac{x}{y} \quad = \frac{x \times x}{x \times y} \quad = \frac{x^2}{xy}$$

❺ The reverse is true, so that a number (including unknowns) that appears on the top and bottom can be cancelled out.

$$\frac{5x}{25} \quad = \frac{5 \times x}{5 \times 5} \quad = \frac{x}{5} \qquad \text{and} \qquad \frac{k^2 + 3k + 2}{4k+4} = \frac{(k+2)\cancel{(k+1)}}{4\,\cancel{(k+1)}} = \frac{k+2}{4}$$

*But you can **only** cancel out common multiplications, you **cannot** cancel out additions.*

$\frac{x+2}{x+3}$ *cannot be simplified, as it has no common **multiplication** factors.*

❻ Dividing fractions is the same as multiplying by the inverse i.e. turned upside down.

$$\frac{3}{x} \div \frac{4}{y} \qquad = \frac{3}{x} \times \frac{y}{4} \qquad \text{and} \qquad \frac{3}{x} \div 5 \qquad = \frac{3}{x} \times \frac{1}{5}$$

❼ If fractions have the same denominator, then you add them by adding the numerators while keeping the same denominator.

$$\frac{y}{2x} + \frac{4}{2x} \qquad = \frac{y+4}{2x}$$

❽ The reverse is true – a fraction can be split into two fractions added together.

$$\frac{x+4}{5} \qquad = \frac{x}{5} + \frac{4}{5}$$

❾ If fractions do not have the same denominators, then they must be converted so that they do have common denominators before you can add them.

$$\frac{2}{x} + \frac{3}{y} \qquad = \frac{2y}{xy} + \frac{3x}{xy} \qquad = \frac{2y + 3x}{xy}$$

Note: this is **not** “cross multiplying”, which requires an equation, i.e. an = sign, to be valid.

❿ Negative signs should be moved from denominator to numerator. Two should be cancelled.

$$\frac{a}{-b} \quad = -\left(\frac{a}{b}\right) \quad = \frac{-a}{b} \qquad \text{and} \qquad \frac{-a}{-b} \quad = \frac{a}{b}$$

Level 2 Power Laws

For Achieved you should to be able to do the following:

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1) Be able to simplify complex algebraic terms raised to a power, including square roots:

A question will read: Simplify $\sqrt{25x^4y^6}$

and you have to give: = $5x^2y^3$

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2) Simplify complex expressions involving variables to various powers by cancelling:

A question will read: Simplify $\frac{(3x^3)^2}{18x^4}$

and you have to give: = $\frac{x^2}{2}$

Merit does not involve any different skills, but could have more difficult fractional powers than ½.

Answers are best given as fractions with positive powers, but decimals and negative powers are acceptable.

Questions with negative and fractional exponents are most common.

BEDMAS still applies, with any term under the line of a √ sign being effectively in brackets.

Basic for Exponents

❶ Any variable can be written as itself to power 1.

$$x = x^1$$

❷ Any variable to power 0 is equal to 1.

$$x^0 = 1$$

❸ The square root of any number can be written as that power to exponent ½.

$$(16x^2)^{0.5} = \sqrt{16x^2}$$

❹ Negative exponents are used to indicate a division by that exponent and **not** that the number is negative.

$$x^{-2} = \frac{1}{x^2}$$

A negative applied to a bracket means the whole bracket is on the bottom line.

$$3(x+2)(x+1)^{-1} = \frac{3(x+2)}{x+1}$$

❺ When exponents are multiplied, the result is the variable to the sum of the two exponents.

$$x^4 \times x^3 = x^{4+3} = x^7$$

❻ When exponents are divided the result is the first exponent minus the second.

$$x^5 \div x^2 = x^{5-2} = x^3 \quad \text{and} \quad \frac{x^2}{x^4} = x^{2-4} = x^{-2} \quad \left(\text{or } \frac{1}{x^2}\right)$$

Complex fractions should be divided into portions of each term, then simplified separately.

$$\frac{6xy^3}{(3xy)^2} = \frac{6}{9}\,\frac{x}{x^2}\,\frac{y^3}{y^2} = \frac{2}{3}\,\frac{1}{x}\,\frac{y}{1} = \frac{2y}{3x}$$

❼ When an exponent is itself raised by an exponent, the result is the product of the exponents.

$$(x^3)^2 = x^{3\times 2} = x^6$$

This also applies to square roots (which are the same as an exponent to ½).

$$\sqrt{x^8} = (x^8)^{½} = x^{8\times ½} = x^4$$

❽ When two numbers inside brackets are raised by an exponent, the result is each number is raised separately.

$$(3xy^3)^2 = 3^2\, y^2\, (x^3)^2 = 9\, x^2\, y^6$$

This also applies to square roots, which are the same as an exponent to ½.

$$\sqrt{9x^4} = \sqrt{9}\,\sqrt{x^4} = 3\,x^2$$

And also applies to fractions as well.

$$\left(\frac{2}{x^2}\right)^3 = \frac{2^3}{(x^2)^3} = \frac{8}{x^6}$$

❾ Exponents can only be added if they are the same unknown raised to the same power. The power does not change.

$4y^2 + 3y^2 = 7y^2$ but $y^2 + y^3$ cannot be simplified

Level 2 Expanding Expressions

For Achieved you should to be able to do the following:

1) Expand complex expressions involving addition or subtraction of multiple terms:

A question will read: Expand and simplify $4\ (3 - 2x) - 3\ (x - 2)$

and you have to give: $= 18 - 11x$

2) Expand complex expressions involving multiplication of three terms:

A question will read: Expand $(x - 2)\ (x - 3)\ (x + 1)$

and you have to give: $= x^3 - 4x^2 + x + 6$

Merit does not require any further skills.

You must add like terms to fully simplify any expansion.

The questions will almost always have a double negative in them. Most mistakes with these questions is poor handling of the negatives.

The order in which you write your answers does not matter, although it is convention to write in decreasing order of the powers (i.e. $ax^3 - bx^2 + cx + d$)

Basics for Expanding

❶ A number or variable outside a set of brackets is multiplied by all of the contents of the bracket.

$4\,(3 - 2x) \quad = 4 \times 3 + 4 \times -2x \quad = 12 - 8x$

❷ Any negative sign outside the brackets is also multiplied into each interior term.

$4 - x\,(a + b) \quad = 4 - ax - bx$

❸ A negative sign by itself outside brackets is the same as multiplying by –1.

$-(a - c) \quad = -a + c$

❹ If two sets of brackets are multiplied together, then each term of the first set of brackets is multiplied by each term of the second set of brackets.

$$(x + y + z)\,(a + b + c) = x\,(a + b + c) + y\,(a + b + c) + z\,(a + b + c)$$
$$= xa + xb + xc + ya + yb + yc + za + zb + zc$$

❺ If a bracket is squared or cubed, then the whole of the bracket is multiplied by itself the appropriate number of times.

$(a + b)^3 \quad = (a + b)\,(a + b)\,(a + b)$

❻ Multiplication is commutative, so the order of brackets does not matter.

$(x + 1)\,(x + 5) \quad = (x + 5)\,(x + 1)$

❼ Multiplying three sets of brackets it is done by multiplying out one pair out first, then afterwards multiplying the result by the remaining one.

$(x + 1)\,(2x + 5)\,(x - 2) \quad = (x + 1)\,(2x^2 + x - 10)$

$= x\,(2x^2 + x - 10) + 1\,(2x^2 + x - 10) \quad = 2x^3 + x^2 - 10x + 2x^2 + x - 10$

$= 2x^3 + 3x^2 - 9x - 10$

(*An answer can be checked by factorising the resulting terms on a graphics calculator, or by substituting in x = 10 and seeing if the answer is the same both ways.*)

Far fewer mistakes are made when students line up like terms in the 2nd multiplication stage and don't try to shortcut the process.

$(2x - 3)\,(x - 2)\,(x + 2) \quad = (2x^2 - 4x - 3x + 6)\,(x + 2)$

$$= x\,(2x^2 - 7x + 6) + 2\,(2x^2 - 7x + 6)$$

$$\begin{array}{rrrrr} = & 2x^3 & -\,7x^2 & +\,6x & \\ + & & 4x^2 & -\,14x & +\,12 \\ \hline = & 2x^3 & -\,3x^2 & -\,8x & +\,12 \end{array}$$

It does not matter which pair of brackets is done first, but it pays to leave the simplest bracket for last, preferably so that all the negatives are dealt with in the first stage (as in the example above).

Level 2 Factorising Expressions

For Achieved you should to be able to do the following:

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1) Factorise a quadratic with an x^2 term greater than 1:

A question will read: Factorise $2x^2 – 3x – 5$

and you have to give: $= (2x – 5)(x + 1)$

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2) Factorise a cubic or similar expression with a common factor that can be removed:

A question will read: Factorise $3x^3 + 17x^2 + 10x$

and you have to give: $= x\ (3x + 2)(x + 5)$

Merit does not require any further skills, although familiarity with two step factorisations might be expected.

As the questions usually have a negative in them, it is easy to make a mistake and put the negative in the wrong brackets. It pays to multiply out your answer in order to check it.

Steps for Factorising Quadratics

❶ Write the quadratic in the form $ax^2 + bx + c$.

The graphics calculator will solve an equation of form $ax^2 + bx + c = 0$.

$12x^2 – 5x – 2 = 0$ has solutions $x = 0.6667$ and $x = – 0.25$

When factorising you don't want those solutions. Instead you need the term x – solution = 0. We get this by placing the **negatives** of those solutions into brackets with the a term in front.

$12x^2 – 5x – 2 = 12 (x – 0.6667) (x + 0.25)$

The leading term is now multiplied into the brackets in such a way to give whole numbers throughout.

$12 (x – 0.6667) (x + 0.25)$

$= 3 (x – 0.6667) \times 4 (x + 0.25)$

$= (3x – 2) (4x + 1)$

It pays to check the result by either substituting in a value (such as $x = 10$) to both start and finish, or by expanding the answer to ensure you get the start terms back.

❷ If the quadratic calculator only gives one solution, then the answer is a square.

$16x^2 – 40x + 25 = 0$ has solution $x = 1.25$

$= 16 (x – 1.25) (x – 1.25)$

$= 4 (x – 1.25) \times 4 (x – 1.25)$

$= (4x – 5)^2$

❸ Don't be tempted to shortcut steps: all quadratics need to be properly factorised.

$x^2 – y^2 \neq (x – y)^2$ because $(x – y)^2 = (x – y) (x – y) = x^2 – 2xy – y^2$

❹ Taking the square root of a squared term gives both a positive and negative answer.

$x^2 = 4 \Rightarrow x = \pm 2$ and $(x – 4)^2 = 9 \Rightarrow x – 4 = \pm 3 \Rightarrow x = 1$ or $x = 7$

❺ If given a cubic expression (i.e. with a x^3 term), the same steps are carried out, as your calculator will factorise cubics as well.

$4x^3 + 20x^2 + 31x + 15$

$= 4 (x + 1) (x + 1.5) (x + 2.5)$ Found by solving $4x^3 + 20x^2 + 31x + 15 = 0$

$= (x + 1) (2x + 3) (2x + 5)$ Multiplying in the 4 to remove the fractions

❻ Potentially other common factors might need to be removed before starting to factorise the standard terms.

$4x^2y – 5xy + 6y$

$= y (4x^2 – 5x + 6)$ then factorise the bracketed part as normal

$= y (4x + 3) (x – 2)$

❼ (Merit) Some expressions require two steps of factorisation. They can be recognised by the way they start as four terms with common factors throughout.

$6ac + 2ad + 3bc + bd = 2a (3c + d) + b (3c + d) = (2a + b)(3c + d)$

Level 2 Solving Equations

For Achieved you should to be able to do the following:

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1) Solve linear equations, especially in the form of algebraic fractions:

A question will read: Solve $\frac{4x + 6}{4} = \frac{x + 10}{5}$

and you have to give: $x = \frac{5}{8}$

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2) Solve linear inequations:

A question will read: Solve $x - 2 > 5x + 8$

and you have to give: $x < -2.5$

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3) Solve quadratic equations:

A question will read: Solve $2x^2 = 3x + 5$

and you have to give: $x = -1$ or $x = 2.5$

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4) Solve linear simultaneous equations:

A question will read: Solve $y = 3x + 5$ and $y = 5x + 4$

and you have to give: $x = 0.5$, $y = 6.5$

Excellence requires understanding of the quadratic formula and the determinant, $\Delta = b^2 - 4ac$, for finding the number of roots of a quadratic.

Solutions can be given in either fractional or decimal form, but must include the full answer (both answers for a quadratic and both x and y for a simultaneous equation).

Basics for Solving

❶ All the techniques from Year 11 are expected, but need to be much more quickly applied. You cannot progress with Year 12 until you are capable of quickly solving simple equations.

❷ To remove a fraction multiply **all** of both sides by the denominator.

$\frac{5x+6}{4} = x + 5 \qquad \Rightarrow\ 5x + 6 = 4\,(x + 5) \qquad \Rightarrow\ 5x + 6 = 4x + 20$ etc

This includes if the bottom line includes variables.

$\frac{2x+3}{4x+7} = x + 1 \qquad \Rightarrow\ 2x + 3 = (4x + 7)\,(x + 1) \qquad \Rightarrow\ 2x + 3 = 4x^2 + 11x + 7$ etc

When both sides are a fraction this process is sometimes called "cross multiplying" but this is the work of the Devil and to be avoided as a rote process. Take all questions involving two fractions on their merits – do **not** be tempted to cross-multiply automatically.

$\frac{x+6}{3} = \frac{2x+1}{5} \qquad \Rightarrow\ 5\,(x + 6) = 3\,(2x + 1) \qquad \Rightarrow\ 5x + 30 = 6x + 3$

❸ Inequations are solved by the same rearrangements as for equations, whether $<, >, \leq$ or $\geq$.

$a - 5 > 16 \qquad \Rightarrow a > 16 + 5 \qquad \Rightarrow a > 21$

The sign of an inequation is swapped if

(a) both sides are multiplied or divided by a negative (**not** just if the answer is negative).

$^{-}10\,x > 15 \qquad \Rightarrow\ x < 15 \div {}^{-}10 \qquad \Rightarrow\ x < {}^{-}1.5$

(b) if two fractions are inverted (rare). You are better to multiply out in these cases.

$\frac{1}{4x} > \frac{2}{5x} \qquad \Rightarrow\ \frac{4x}{1} < \frac{5x}{2}$

(c) when the sides of the equation are swapped.

$a + 7 > b \qquad \Rightarrow\ b < a + 7$

If care is taken, it is almost always possible to solve an inequation so that no negative terms arise and the sign never needs to be swapped.

$3x > 4x - 15 \qquad \Rightarrow\ 0 + 15 > 4x - 3x \qquad \Rightarrow\ 15 > x$

❹ Quadratic equations need to be recognised as such, then solved on the calculator (in "Equations" then "Polynomial" and the "Degree" is the highest power). Care is needed to rearrange the equation to the correct format first.

$2x^2 + 3x = 20 \qquad \Rightarrow\ 2x^2 + 3x - 20 = 0 \qquad \Rightarrow\ :\ x = -4$ or $x = 2.5$

❺ Simultaneous linear equations can be solved by rewriting the equations to the appropriate form and solving on the calculator.

$y = 3x + 5$ and $y = 5x + 4$ needs to be formed as $-\,3x + y = 5$ and $-\,5x + y = 4$

❻ (Merit) When asked to simplify an complex fraction with a quadratic term, there will always be a common factor. But when asked to solve a similar looking situation there won't be a common term, so you need to multiply across the bottom term rather than try to cancel first.

$\frac{x^2+5x+3}{4x+7} = 4 \qquad \Rightarrow\ x^2 + 5x + 3 = 4(4x + 7)$ etc

Level 2 Logarithms

For Achieved you should to be able to do the following:

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1) Solve simple equations based on the use of " If $y = b^x$ then $x = \log_b(y)$ ":

A question will read: Solve $\log_x(64) = 2$

and you have to give: $x = 8$

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2) Simplify terms using the three log rearrangements given:

A question will read: Simplify to one term $\log_b(xy) - 3\log_b(x)$

and you have to give: $= \log_b\left(\frac{y}{x^2}\right)$

Basics for Logarithms

❶ For any question of the type "Solve $x = \log_b(y)$" you rearrange the unknowns to remove the log by using the relationship "If $y = b^x$ then $x = \log_b(y)$" – given on the formula sheet.

$\log_x(64) = 6 \quad \Rightarrow 6 = \log_x(64) \quad \Rightarrow 64 = x^6 \quad \Rightarrow x = \sqrt[6]{64} \quad \Rightarrow x = 2$

$x = \log_b(y) \qquad y = b^x$

$\log_5(x) = 4 \quad \Rightarrow 4 = \log_5(x) \quad \Rightarrow x = 5^4 \quad \Rightarrow x = 625$

$x = \log_b(y) \qquad y = b^x$

Note: you need to know how to use the power (^ or y^x) and root ($^x\sqrt{}$) buttons on your calculators in order to solve these.

❷ For questions asking you to simplify several log terms into one log term, there are three rules.

1) A multiplier in front becomes a power inside the log. (Note this one is given on the formula sheet, but the other two are not.)

   $\log(x^n) = n \log(x)$

2) Any additions on the outside become multiplications inside the log term.

   $\log(x) + \log(y) = \log(xy)$

3) Any subtractions on the outside become divisions inside the log term.

   $\log(x) - \log(y) \quad = \log\left(\frac{x}{y}\right)$

❸ It is important to use remove the multipliers in front as the first step in any simplification.

$2\log_b(a) - \log_b(b) \quad = \log_b(a^2) - \log_b(b) \quad = \log_b\left(\frac{a^2}{b}\right)$

❹ Be careful to apply any use of "$\log_b(x^n) = n\log_b(x)$" to the **whole** term inside the bracket.

$2\log_b(4x) \quad = \log_b((4x)^2) \quad = \log_b(16x^2)$

❺ Make sure that any terms inside the final brackets are fully simplified in your answer.

$3\log_b(a) - 2\log_b(2a) \quad = \log_b(a^3) - \log_b((2a)^2) \quad = \log_b(a^3) - \log_b(4a^2)$

$\quad = \log_b\left(\frac{a^3}{4a^2}\right) \quad = \log_b\left(\frac{a}{4}\right)$

This applies to numerical examples too, where any squares, divisions and multiplications must be completed.

$\log_b(12) - 2\log_b(2) \quad = \log_b(12) - \log_b(2^2) \quad = \log_b(^{12}/_4) \quad = \log_b(3)$

❻ You can only calculate the final numerical value of a log if you know the base it is in. An answer of form "$\log_b(5)$" cannot be simplified until you know the base, b. Most log manipulation questions will therefore end up with the answer still in log form.

Your calculators only give logs to base 10 ("log" button) and base e ("ln" button).

❼ (Merit) To calculate a log value in a known base, it helps to memorise the formula for changing them into base 10, so they can be done quickly on the calculator.

$\log_b(a) = \frac{\log_{10}a}{\log_{10}b}$

Level 2 Exponential Equations

For Achieved you should to be able to do the following:

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1) Solve the simplest exponential equations:

A question will read: Solve $7 = 3^x$

and you have to give: $x = 1.771$

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2) Put values into more complex equations and interpret the results in context:

A question will read: If a value is given by $V = 200 \times (1.09)^t$ find the value at 5 years

and you have to give: V = $307.72

Merit includes the ability to solve equations of the form $8 = 5 \times 4^x$ by algebraic methods.
You should also be able to form equations for exponential growth of the form:

$$\text{end} = \text{start} \times \text{rate}^{\text{time}}$$

and recognise what each portion of the equation does.

Merit includes the ability to write and solve equations of the form $8 = 5 \times 4^x$, often from context.

Basics for Exponential Equations

❶ Any problem where the unknown is an exponent will generally have to be solved using logs.

❷ If it is an equation where the only unknown is a power, then by taking the log of both sides of an equation and then applying $\log(x^n) = n \log(x)$, one removes the exponent part of the equation, making it solvable using normal linear algebra.

$4^x = 14 \quad \Rightarrow \log(4^x) = \log 14 \quad \Rightarrow x \log(4) = \log(14)$

$\Rightarrow x = \log(14) \div \log(4) \quad \Rightarrow x = 1.90$

❸ (Merit) Equations of the form $8 = 5 \times 4^x$ are solved by the same method with one more step, basically removing the extra term.

$8 = 5 \times 4^x \quad \Rightarrow \frac{8}{5} = 4^x \quad \Rightarrow \log_{10}(1.6) = \log_{10}(4^x)$

$\Rightarrow \log(1.6) = x \log(4) \quad \Rightarrow x = \log(1.6) \div \log(4) = 0.339$

They can also be solved by a related method, using log manipulation.

$8 = 5 \times 4^x \quad \Rightarrow \log_{10}(8) = \log_{10}(5 \times 4^x) \quad \Rightarrow \log(8) = \log(5) + x \log(4)$

$\Rightarrow x = (\log 8 - \log 5) \div \log 4 \quad \Rightarrow x = 0.339$

❹ (Merit) You may be asked to write an exponential equation first, before solving. They can be simplified to the general form of:

end value = start value × (1 + change)$^{\text{time}}$

The change has to be expressed as a decimal, although often given as a % in the question. Any losses are negative changes.

e.g. If a $20,000 car loses 5% per year, the start value is 20,000 and the change is –0.05

value = 20,000 × $(1 - 0.05)^t$ which can be simplified inside the bracket

value = 20,000 × 0.95^t

❺ (Excellence) If the amount to be calculated is for a change in value from a period which is not that of t = 0, then a more complicated equation emerges.

e.g. the change in value for the car above between time 1 and time 2 is

change = value at time 1 – value at time 2

$= 20{,}000 \times (1 - 0.05)^{t1} - 20{,}000 \times (1 - 0.05)^{t2}$

$= 20{,}000 \times [(1 - 0.05)^{t1} - (1 - 0.05)^{t2}]$

$= 20{,}000 \times (1 - 0.05)^{t1} \times (1 - 0.05)^{t2 - t1}$

(because $x^a - x^b = x^a(1 - x^{b-a})$ as we are factoring out the common x^a term)

or instead it can be rearranged as

change $= 20{,}000 \times (1 - 0.05)^{t1} \times (0.05 - 1)^{t1 - t2}$

(because $x^a - x^b = x^b(x^{b-a} - 1)$ as we are factoring out the common x^b term)

Level 2 Using the Quadratic Formula

For Achieved you should to be able to do the following:

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1) Find the solutions to a quadratic by hand using the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

A question will read: Solve $2x^2 + 3x - 2 = 0$ using the quadratic formula

and you have to give: $x = 0.5$ and -2

Merit includes recognising that a situation would need expanding or simplification first, to get a suitable form for solving e.g. solve $x^2 + 4x + 6 = 5x^2 + 2x + 17$.

Excellence includes the ability to do this with one or more of the parameters a, b or c given as a variable e.g. solve $x^2 + kx + k^2 = 0$.

Excellence students must also be able to apply the discriminant, $b^2 - 4ac$, to solving quadratics.

Basics for Quadratic Formula

❶ For any equation of the form $x^2 + b\,x + c = 0$ the solutions are found by $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$:

The ± means there are two solutions, one positive and one negative.

e.g. to solve $3x^2 - 5x + 2 = 0$ $\quad a = 3,\ b = -5,\ c = 2$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{--5 \pm \sqrt{(-5)^2 - 4\times3\times2}}{2\times3}\ x = \dfrac{5 \pm \sqrt{1}}{6}$

so $x = \dfrac{-5+1}{6}$ and $x = \dfrac{5-1}{6}$ $\quad x = 1$ and 👌

❷ (Merit) The same procedure is followed if the parameters happen to be unknowns themselves.

e.g. to solve $6x^2 + 5kx + k^2 = 0$ $\quad a = 6,\ b = 5k,\ c = k$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-5k \pm \sqrt{(5k)^2 - 4\times6\times k^2}}{2\times6}\ x = \dfrac{-5\mathrm{k} \pm \sqrt{k^2}}{12}$

so $x = \dfrac{-5k+k}{12}$ and $x = \dfrac{-5k-k}{12}$ $\quad x =$ ⁻🗎k and ⁻🗁k

❸ (Excellence) A quadratic in the form $ax^2 + bx + c = 0$ has two real roots if the discriminant, $\Delta = b^2 - 4ac$, is positive, one if $\Delta = 0$ and none if Δ is negative.

e.g. we know the quadratic $3x^2 + 18x + 27$ has only one root, as $18^2 - 4 \times 3 \times 27 = 0$

❹ (Excellence) Most often we need to work the process in reverse, so that the value of the discriminant is given and the value of some unknown has to be calculated.

$x^2 + kx + 23$ has one real root when $k^2 - 4 \times 1 \times 23 = 0$, so when $k^2 = 92$

$k = \pm\sqrt{92}$

Be careful with these to make sure you take the negative as well as the positive roots.

❺ (Excellence) The nature of the discriminant can be tested in various ways:

a) the number of real roots is two if $\Delta > 0$,

b) if $\Delta = 0$, then the quadratic is a perfect square (i.e. has only one root),

c) the quadratic cannot be factorised is $\Delta < 0$,

d) the graph of a quadratic does not intersect the x-axis if its $\Delta < 0$,

e) a tangent will intersect a curve such that the solution to the simultaneous equations for x or y gives $\Delta = 0$,

f) two graphs do not meet if the solution to the simultaneous equations for x or y gives $\Delta < 0$.