L2 Calculus Revision #6

1. What does it mean that the gradient function of $f(x) = x^3 + 2x^2 + 7$, has a value of 64 at $x = 4$?

2. Show that the tangents at (2, −38) and (−2, 50) on the function $k(x) = 3x^3 - 34x + 6$ are parallel.

3. If tensile strength is given by $\text{TS} = 1500 - 3t^2$ (where strength is in kilo-pascals, kPa, and t is time in hours), how is the tensile strength changing by the fourth hour?

4. Find the equation of the curve going through the point (2, 4) which has its gradient at any point given by $\frac{dy}{dt} = 5x^2 + 4$.

5. An electron in an electric field has its distance given by $s = 3t^2 - 2.1t$
   where s is the distance from a fixed point A in μm and t is the time in milliseconds (ms).

   When is the electron's distance from point A not changing?

6. The velocity of particle (in m s^{-1}) is given by the function: $v(t) = -4t^2 + 20t + 12$

   What is the highest speed in the positive direction that the particle reaches?

   What is the highest speed in the negative direction that the particle reaches?

Answers: L2 Calculus Revision #6

1. It means that **when the function is graphed the gradient is 64 at the point (4, 103).**

   Alternatively: **the rate of change of y relative to x is 64 when x = 4.**

2. $k(x) = 3x^3 - 34x + 6$ so $k'(x) = 9x^2 - 34$

   $k'(2) = 9 \times (2)^2 - 34 = 2$ and $k'(-2) = 9 \times (-2)^2 - 34 = 2$

   Both points have a gradient of 2, so their tangents also have gradients of 2.

   Lines with the same gradient are parallel.

3. $\mathrm{TS} = 1500 - 3t^2$ so $\frac{d\mathrm{TS}}{dt} = -6t$

   When $t = 4$, so $\frac{d\mathrm{TS}}{dt} = -6 \times 4 = -24$ The negative means it is getting less.

   The tensile strength is **dropping** by **24 kPa per hour**

4. $\frac{dy}{dt} = 5x^2 + 4$ so $y = {}^5/_3\, x^3 + 4x + \mathrm{C}$ (2, 4) means $f(2) = 4$

   so $4 = {}^5/_3 \times 2^3 + 4 \times 2 + \mathrm{C}$ so $\mathrm{C} = -21.333$ $\boldsymbol{y = \frac{5}{3}x^3 + 4x - 17\frac{1}{3}}$

5. $s(t) = 3t^2 - 2.1t$ so $v(t) = 6t - 2.1$ Distance not changing when $v = 0$

   $6t - 2.1 = 0$ when $t = 0.35$ After **0.35 ms**

6. $v(t) = -4t^2 + 20t + 12$ $v'(t) = -8t + 20$ Max/min when derivative = 0

   $v'(t) = -8t + 20 = 0$ solving gives $t = 2.5$

   $v(2.5) = -4 \times (2.5)^2 + 20 \times (2.5) + 12 = 37$ A + value, so must be + maximum

   Maximum positive velocity = 37 ms^{-1}

   The **maximum velocity in the negative direction cannot be calculated by calculus**. Theoretically the equation says it will be infinite. Ignoring negative time, the greatest negative velocity will be at the last time the equation applies for.

Questions 5 and 6 are Merit