L2 Calculus Practice #2

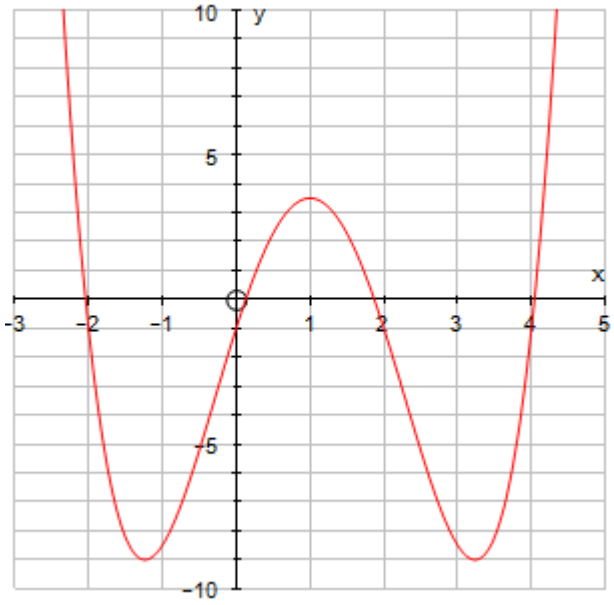
1. The gradient at any point on a curve is given by: $\frac{dy}{dx} = \frac{3x^2}{4} + 1$

   The curve passes through the point (4, 10).

   Find the equation of the curve.

2. To the right is a graph of a function, $f(x)$.

   Clearly indicate the portions where $f'(x) \geq 0$.



3. Find the gradient at (4, 5) of the function $f(x) = 0.25x^2 - x + 5$.

4. Where on the graph of $y = 2x^2 + 5x + 9$ would the tangents to the curve have the form: $y = 2x + c$?

5. Find the coordinates of the turning points of the graph of $f(x) = 2x^3 + 6.5x^2 - 5x + 4$ and determine their nature.

6. A ball starts at 0 seconds with a positive velocity, and slows until it stops after 6 seconds.

   The acceleration is given by: $a = -0.4t$ (where t is time, in seconds)

   Give the equation for the velocity of the ball.

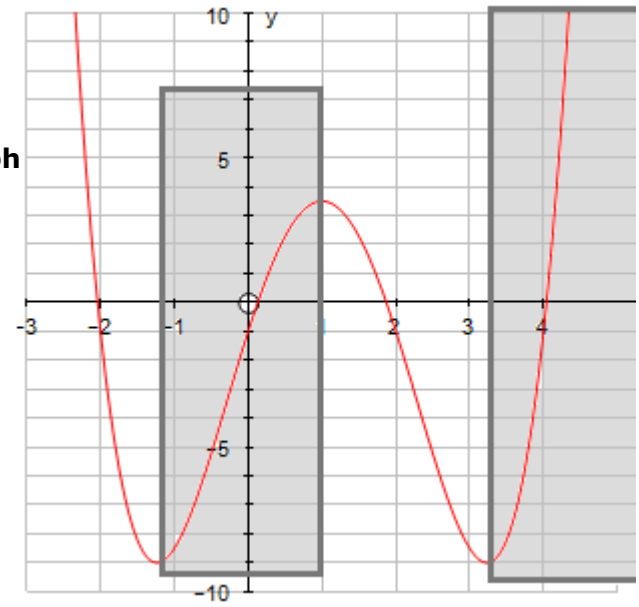
Answers: L2 Calculus Practice #2

1. $\frac{dy}{dx} = \frac{3}{4}x^2 + 1$ so $y = ¾\ x^2 + 1 + C$

   Passes through (4, 10) so $10 = ¼ \times 4^3 + 4 + C$. Solving gives C = −10

   Equation is $y = ¼\ x^3 + x - 10$

2. **Shown how it might be done on the graph**



   Note: the right hand area should be shown as being infinite to the right.

   Formally: $-1.2 \geq x \geq 1$ and $x \geq 3.3$

   Note: any method indicating the correct portion is acceptable.

3. $f(x) = 0.25x^2 - x + 5$ so $f'(x) = 0.5x - 1$

   We want gradient at $x = 4$ $f'(4) = 0.5 \times 4 - 1 = 1$ **Gradient = 1**

4. Tangents have the form $y = 2x + c$ wherever the graph itself has a gradient of 2.

   $y = 2x^2 + 5x + 9$ so $\frac{dy}{dx} = 4x + 5 = 2$ $x = -0.75$ **at (−0.75, 6.375)**

5. $f(x) = 2x^3 + 6.5x^2 - 5x + 4$ so $f'(x) = 6x^2 + 13x - 5$

   Turning points are at $f'(x) = 0$ so we solve $0 = 6x^2 + 13x - 5$

   $0 = (3x - 1)(2x + 5)$ or calculator gives solutions of $x = ⅓$ and $x = -2½$

   $f(⅓) = 3.13$ and $f(-2.5) = 25.875$ and checking graph for their nature

   Maximum at (−2.5, 25.875) **Minimum at (0.333, 3.13)**

6. $a = -0.4t$ so $v = -0.2t^2 + C$

   Stops at t = 6, so $0 = -0.2 \times 6^2 + C$, which gives C = 7.2

   $v = 7.2 - 0.2t^2$ **(or $v = -0.2t^2 + 7.2$)**

Questions 5 and 6 are Merit