L2 Calculus Practice #4

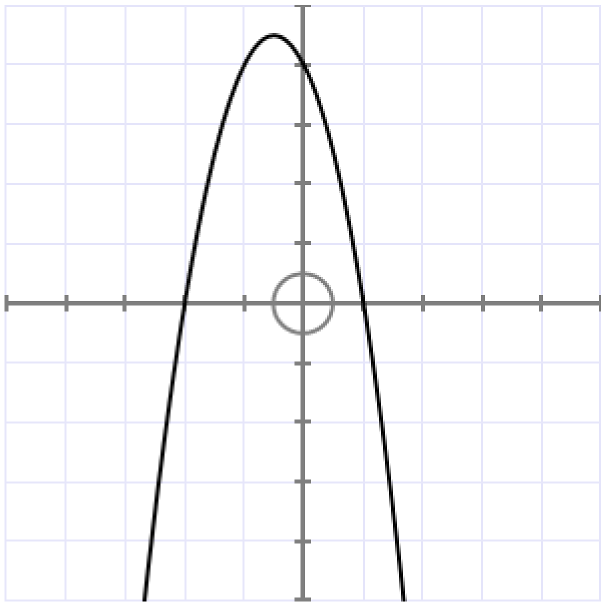
1. Find the gradient of $f(x) = 2x^3 - 4x^2 + 5$ at $x = 3$.

2. The gradient at any point on a curve is given by $\frac{dy}{dx} = 5x - 4$

   Find the equation of the curve if it passes through the point (2, 4).

3. To the right is a gradient function.

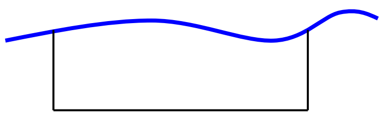
   Sketch a function that would generate such a gradient function



4. A parabola has equation $y = 5 - 2x^2 + 4x$.

   Find the point where the slope of the curve is 2

5. Find the maximum area of a rectangular field that can be made with 48 metres of fencing if one side is alongside a stream and does not need to be fenced. You may assume the stream is basically straight.



6. The acceleration of an object is steady at 5 ms^{-2}.

   At the start time and distance ($t = 0$, $s = 0$) the object is moving at 10 ms^{-1}.

   Find how far the object travels in the first 5 seconds.

Answers: L2 Calculus Practice #4

1. $f(x) = 2x^3 - 4x^2 + 5$ so $f'(x) = 6x^2 - 8x$

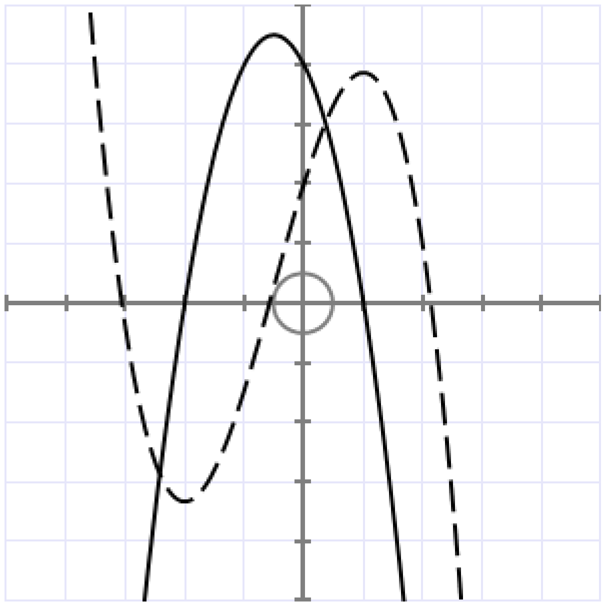
   gradient is found from gradient function $f'(3) = 6 \times 3^2 - 8 \times 3 = 30$

   Gradient at x = 3 is 30

2. $\frac{dy}{dx} = 5x - 4$ anti-differentiating gives $y = 2.5x^2 - 4x + C$

   Passes through (2, 4) so $4 = 2.5 \times 2^2 - 4 \times 2 + C$ So C = 2

   Equation is $y = 2.5x^2 - 4x + 2$



3. **A negative cubic as shown**, where the minimum is on the parabola's left intercept and the maximum at the parabola's right intercept.

   (The cubic can intercept the axes at any points.)

4. $y = 5 - 2x^2 + 4x$, so $\frac{dy}{dx} = -4x + 4$

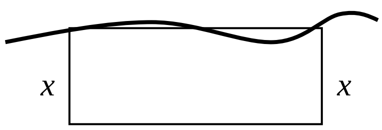
   so $\frac{dy}{dx} = -4x + 4$

   We want gradient = 2 so $\frac{dy}{dx} = 2 = -4x + 4$

   which is when $x = 0.5$.

   Need the point, so sub into original equation. **Point at (0.5, 6.5)**

5. Area = base × height = $x(48 - 2x) = 48x - 2x^2$



   $\frac{dy}{dx} = 0$ at maximum

   $\frac{dy}{dx} = 48 - 4x = 0$ so when $x = 12$

   Area = 12 × (48 − 2 × 12) **The maximum area = 288 m²**

6. a = 5 ms^{-2} so $v = \int 5.\square\square\, dt = 5t + C$ At $t = 0$, v = 10 ms^{-1} so $v = 5t + 10$

   $s = \int 5t + 10.\,dt\square\square = 2.5t^2 + 10t + C$ At $t = 0$, s = 0 $s = 2.5t^2 + 10t$

   At time = 5, $s = 2.5 \times 5^2 + 10 \times 5$ **Distance travelled = 112.5 m**

Questions 5 and 6 are Merit