L2 Calculus Practice #6

1. Find the gradient function of $f(x) = 4x^3 + 1.2x^2 + 17$, and explain what that tells you.

2. What point on $f(x) = 3x^2 - 12x + 6$ has a slope of 6?

3. If volume (in m^3) is given by $V = 2t^3 + 3t^2 - 12t + 60$ (where t is time in minutes), find the rate of change of Volume at the second minute.

4. Find the equation of the curve going through the point (1, 4) which has a gradient function $f'(x) = 10x^4 + 4$.

5. A particle has a velocity function given by $v = t - 0.2$
   where v is the rate of change of distance from a fixed point A in m s^{-1} and t is the time after release in seconds.

   What is the distance of the particle from A after 3 seconds, if it started 20m from A?

6. The displacement of particle A (in m) is given by the function: $s_A(t) = \frac{2}{3}t^3 + \frac{5}{2}t^2 + 2t$

   The velocity of particle B (in m s^{-1}) is given by the function: $v_B(t) = t^2 + 8t + 12$

   Particles A and B move from the same point in the same direction at $t = 0$ seconds.

   What is the distance between particles A and B when their velocities are equal?

Answers: L2 Calculus Practice #6

1. $f(x) = 4x^3 + 1.2x^2 + 17$ so $\boldsymbol{f'(x) = 12x^2 + 2.4x}$

   The new function allows us to **find the gradient (or rate of change) at any point** on the original function by substituting in the same value of x for that point.

2. $f(x) = 3x^2 - 12x + 6$ so $f'(x) = 6x - 12$

   We need when $f'(x) = 6$ $\quad (f'(x) =)\ 6x - 12 = 6$ $\quad x = 3$

   Asked for a point, so find $f(3) = \mathbf{-3}$ The point is **(3, –3)**

3. $V = 2t^3 + 3t^2 - 12t + 60$ so $\frac{dV}{dt} = 6t^2 + 6t - 12$

   Want rate of change = gradient, when $t = 2$, so $\frac{dV}{dt} = 6(2)^2 + 6 \times (2) - 12 = 24$

   The rate of change is 24 $\mathbf{m^3s^{-1}}$

4. $f'(x) = 10x^4 + 4$ so $f(x) = 2x^5 + 4x + C$ $\quad (1, 4)$ means $f(1) = 4$

   so $4 = 2 \times 1^5 + 4 \times 1 + C$ so $C = -2$ $\quad \boldsymbol{f(x) = 2x^5 + 4x - 2}$

5. $v = t - 0.2$ so $s(t) = 0.5t^2 - 0.2t + C$ $\quad s(0) = 20$ so $C = 20$

   $s(3) = 0.5 \times 3^2 - 0.2 \times 3 + 20 = 23.9$ Distance = **23.9 m**

6. Velocities equal when $v_B(t) = v_A(t)$ and $v_A(t) = s'_A(t) = 2t^2 + 5t + 2$

   So $2t^2 + 5t + 2 = t^2 + 8t + 12$ gives $t^2 - 3t - 10 = 0$

   solving $t = 5$ or -2 we can ignore negative time, so $t = 5$

   $s_A(5) = \frac{2}{3}5^3 + \frac{5}{2}5^2 + 2 \times 5 = 155.833$

   $v_B(t) = t^2 + 8t + 12$ so $s_B(t) = \frac{1}{3}t^3 + 4t^2 + 12t + C$ $\quad s_B(0) = 0$, so $C = 0$

   $s_B(5) = \frac{1}{3}5^3 + 4 \times 5^2 + 12 \times 5 = 201.666$

   Difference = $s_B(5) - s_A(5) = 201.666 - 155.8333 =$ **45.8333 m**

Questions 5 and 6 are Merit