L2 Calculus Revision #2

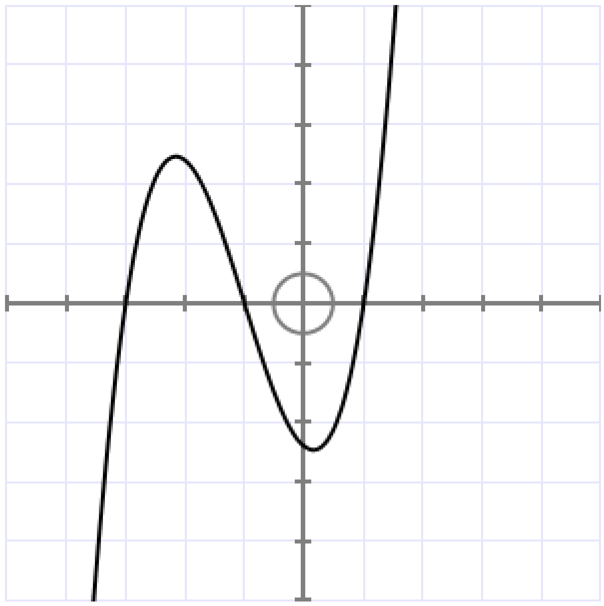
1. The gradient at any point on a curve is given by: $\frac{dy}{dx} = 2x^3 - 9$

   The curve passes through the point (4, 10).

   Find the equation of the curve.

2. To the right is a graph of a function, $f(x)$.

   Sketch on the grid its gradient function.



3. Find the gradient at (5, 3) of the function $f(x) = 0.2x^2 - 2x + 8$.

4. The quantity of water in a tank is given by L = $100 + 60t - 2t^2$ (where L is in metres, and t is in hours). How fast is the level changing after 3 hours?

5. Find the coordinates of the turning points of the graph of $f(x) = 1\frac{2}{3}x^3 + 24x^2 - 20x + 1$ and determine their nature.

6. The distance in metres of a ball from a point is given by $s(t) = 8 - 4t + 4t^2$

   (where t is time, in seconds)

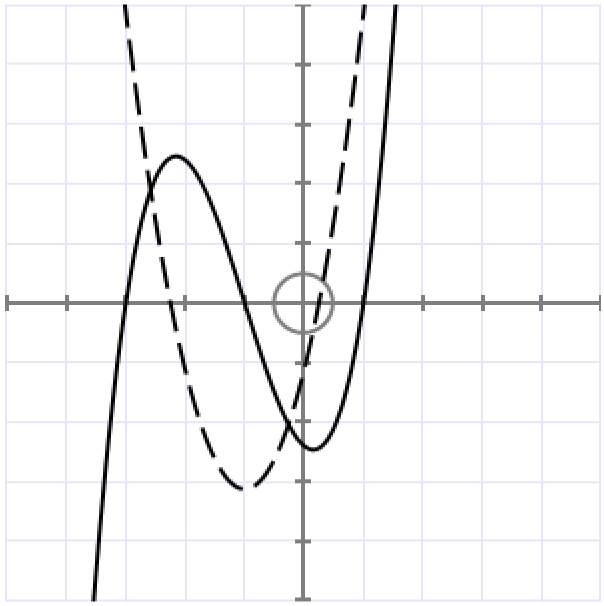
   Give the equation for the velocity of the ball, and show how far it is away from the point when it first comes to a stop.

Answers: L2 Calculus Revision #2

1. $\frac{dy}{dx} = 2x^3 - 9$ so $y = 0.5x^4 - 9x + C$

   (4, 10) on line so $10 = 0.5 \times 4^4 - 9 \times 4 + C$ $C = -82$

   Equation is $y = ½ x^4 - 9x - 82$



2. **Drawn as dotted line**

   Any positive parabola provided that the intercepts are at the turning points of the cubic.

3. $f(x) = 0.2x^2 - 2x + 8$ so $f'(x) = 0.4x - 2$

   We want gradient at $x = 5$ $f'(5) = 0.4 \times 5 - 2 = 0$ **Gradient = 0**

4. $L = 100 + 60t - 2t^2$ $L' = 60 - 4t$ At $t = 3$, $L' = 60 - 4 \times 3 = 48$

   The level is changing at 48 metres per hour

5. $f(x) = 1\frac{2}{3}x^3 + 24x^2 - 20x + 1$ so $f'(x) = 5x^2 + 48x - 20$

   Turning points are at $f'(x) = 0$ so we solve $0 = 5x^2 + 48x - 20$

   $0 = (5x - 2)(x + 10)$ or calculator gives solutions of $x = 0.4$ and $x = -10$

   $f(0.4) = -3.053$ and $f(-10) = 934\frac{1}{3}$ and checking graph for their nature

   Maximum at $(-10, 934\frac{1}{3})$ Minimum at $(0.4, -3.053)$

6. $s(t) = 8 - 4t + 4t^2$ so **$v(t) = -4 + 8t$**

   Stops when $v = 0$ when $0 = -4 + 8t$ so when $t = 0.5$

   $s(0.5) = 8 - 4 \times 0.5 + 4 \times (0.5)^2 = 7$ It first stops **7 m** away from the point.

Questions 5 and 6 are Merit