

Co-ordinate Geometry : Merit/Excellence Practice #2

1. $A = (1, 2)$, $B = (9, 5)$, $C = (k, 11)$ are on the same line. Find k .
2. $X = (1, 2)$, $Y = (5, 3)$, $Z = (-1, 10)$. Show the triangle XYZ is right-angled.
3. Find a and b so that the lines $y = ax + b$ and $y = 2ax - b$ meet at $(-2, -9)$.
4. How close does the line $y = 2x + 5$ come to the point $(8, 8)$?

Answers – Co-ordinate Geometry : Merit/Excellence Practice #2

1. A = (1, 2), B = (9, 5), C = (k, 11) are on the same line. Find k.

$$m_{AB} = \frac{5-2}{9-1} = \frac{3}{8} = 0.375$$

$$y - 2 = 0.375(x - 1)$$

$$y = 0.375x + 1.625 \quad = \text{equation for AB}$$

$$\text{for C to be on the line } 11 = 0.375 \times k + 1.625$$

$$\mathbf{k = 25}$$

(Alternatively, can also use that the slopes $m_{AB} = m_{AC}$)

2. X = (1, 2), Y = (5, 3), Z = (-1, 10). Show the triangle XYZ is right-angled.

$$m_{XY} = \frac{3-2}{5-1} = \frac{1}{4} = 0.25$$

$$m_{XZ} = \frac{10-2}{-1-1} = \frac{8}{-2} = -4$$

Lines are perpendicular if $m^\perp = \frac{-1}{m}$, which is what we have here, as $-4 = \frac{-1}{0.25}$

Perpendicular lines mean a right angle triangle.

3. Find a and b so that the lines $y = ax + b$ and $y = 2ax - b$ meet at (-2, -9).

$$\text{lines } y = ax + b \text{ and } y = 2ax - b \text{ meet when } ax + b = 2ax - b$$

$$\text{rearranging this gives when } 2b = ax$$

$$\text{substituting this into } y = ax + b, \text{ we get } y = 2b + b, \text{ so } y = 3b$$

$$\text{as the point } (-2, -9) \text{ is on this line, } -9 = 3b, \text{ so } b = -3.$$

$$\text{if } b = -3, \text{ then as } y = ax + b \text{ we find } -9 = a \times -2 + -3, \text{ so } a = 3$$

$$\mathbf{a = 3, b = -3}$$

4. How close does the line $y = 2x + 5$ come to the point (8, 8)?

Nearest point is at 90° , so need perpendicular line: $y = 2x + 5$ has slope 2

$$\text{So perpendicular line has slope} = \frac{-1}{2} = -0.5$$

$$\text{It has equation: } y - 8 = -0.5(x - 8) \Rightarrow y = -0.5x + 12$$

Solve where crosses $y = 2x + 5$ using simultaneous equations, gives (2.8, 10.6)

$$\text{Distance} = \sqrt{(8-2.8)^2 + (8-10.6)^2} = \mathbf{5.81}$$