Co-ordinate Geometry : Merit / Excellence Practice #6

1. E = (1, 4), F = (3, 13), G = (12, 15), H = (10, 6). Show that EFGH is a rhomboid.

2. E = (1, 4), F = (3, 13), G = (12, 15), H = (10, 6). Show that EG is a perpendicular bisector of FH.

3. U = ($^-2$, 1), V = (2.5, 7), W = (4, $^-3.5$). Show the triangle is isosceles.

4. X = (1, 2), Y = (9, 0), Z = (8, 6). Find the equation of the altitude, if XY is the base.

Answers – Co-ordinate Geometry : Merit / Excellence Practice #6

1. E = (1, 4), F = (3, 13), G = (12, 15), H = (10, 6). Show that EFGH is a rhomboid.

   Length EF = $\sqrt{(3-1)^2 + (13-4)^2}$ = $\sqrt{85}$ = Length FG = $\sqrt{(12-3)^2 + (15-13)^2}$

   = Length GH = $\sqrt{(10-12)^2 + (6-15)^2}$ = Length EH = $\sqrt{(10-1)^2 + (6-4)^2}$

   All four lengths are the same, so it must be a rhomboid.

2. E = (1, 4), F = (3, 13), G = (12, 15), H = (10, 6). Show that EG is a perpendicular bisector of FH.

   $m_{\text{EG}} = \dfrac{15-4}{12-1} = \dfrac{11}{11} = 1$ $\qquad$ $m_{\text{FH}} = \dfrac{6-13}{10-3} = \dfrac{-7}{7} = {}^{-}1$

   $m_{\text{EG}} \times m_{\text{FH}} = {}^{-}1$ so the lines are perpendicular

   Midpoint EG = $\left(\dfrac{1+12}{2}, \dfrac{4+15}{2}\right)$ = (6.5, 8.5) $\quad$ Mid FH = $\left(\dfrac{3+10}{2}, \dfrac{13+6}{2}\right)$ = (6.5, 8.5)

   As the midpoints are the same, the lines both bisect each other

   (Can also be answered by finding the equation of EG is $y = x + 3$ and of FH is $y = -x + 16$, then that those meet at (6.5, 8.5). From there show it is the midpoint of FH or that the distance from the intersection to F and H is the same.)

3. U = (${}^{-}2$, 1), V = (2.5, 7), W = (4, ${}^{-}3.5$). Show the triangle is isosceles.

   Length UV = $\sqrt{(-2-2.5)^2 + (1-7)^2}$ = $\sqrt{56.25}$ = 7.5

   Length UW = $\sqrt{(-2-4)^2 + (1--3.5)^2}$ = $\sqrt{56.25}$ = 7.5

   Equal side lengths, so we have an isosceles triangle

4. X = (1, 2), Y = (9, 0), Z = (8, 6). Find the equation of the altitude, if XY is the base.

   $m_{\text{XY}} = \dfrac{2-0}{1-9} = \dfrac{2}{-8} = {}^{-}0.25$

   The altitude is perpendicular to this, so will have $m^{\perp} = \dfrac{-1}{-0.25} = 4$

   and it will pass through point Z. $\qquad$ So $y - 6 = 4(x - 8)$

   $y = 4x - 26$